

國立臺灣師範大學 106 學年度學士班二年級轉學生招生考試試題

科目：微積分

適用學系（組）：數學系

- 注意：1. 本試題共 1 頁，請依序作答，並標明題號，不必抄題。
2. 答案必須寫在答案卷上之指定作答區內，否則依規定予以扣分。
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1. Evaluate the following integrals: (8 points each)

(1) $\int_0^1 (x+1)\sqrt{1-x} dx$

(2) $\int e^{2x} \sin(3x) dx$

(3) $\int_0^1 x \ln x dx$

(4) $\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3) dx dy$

2. If $\int_0^{x^2} f(2t^3 - t^2 - 6) dt = \ln x$, find the value of $f(106)$. (10 points)

3. Find the equation of the tangent line to the graph $x + y - 1 = \ln(x^2 + y^2)$ at point $(1, 0)$. (8 point)

4. If $0 \leq x, y \leq \frac{\pi}{2}$, prove that $\left| \frac{\sin(x-y)}{\cos x \cos y} \right| \geq |x-y|$. (10 points)

5. Determine whether the infinite series $\sum_{n=1}^{\infty} \frac{[(n+1)!]^n}{2!4! \cdots (2n)!}$ is convergent or not. (10 points)

6. Find the relative extrema and saddle point of the function $f(x, y) = 2xy - \frac{1}{2}(x^4 + y^4) + 1$ where $(x, y) \in \mathbb{R} \times \mathbb{R}$. (10 points)

7. Find the area of the surface given by $f(x, y) = 13 + x^2 - y^2$ over the region $\{(x, y) : x^2 + y^2 \leq 4\}$. (10 points)

8. Show that any tangent plane to the cone $z^2 = a^2x^2 + b^2y^2$ passes through the origin. (10 points)