

微積分（乙）期中考 （2014 年 4 月 16 日）

注意事項：1 至 7 大題中每個小題 5 分計算，共 20 個小題，總分 100 分。

1. Determine if the following series converge or diverge, and explain your answer.

$$(1) \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3} \quad (2) \sum_{n=1}^{\infty} (-1)^n (\sqrt{n^2 + n} - n)$$

2. Find the (1) radius of convergence (收斂半徑) and (2) interval of convergence (收斂區間) for the power series $\sum_{n=1}^{\infty} \frac{(x+4)^n}{n3^n}$.

3. (1) Find the Taylor series for $f(x) = e^{-2x}$ at $x = 0$.
(2) Write out the remainder $R_n(x)$ of order n for the function f in (1) in Taylor's formula.
(3) Prove that the Taylor series of f in (1) converges to $f(x)$ for all $x \in (-\infty, \infty)$.

4. Given a parametric curve $x = \sqrt{3}t^2$, $y = 3t - \frac{1}{3}t^3$, $-3 \leq t \leq 3$.

- (1) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.
(2) Find the area enclosed by the curve.
(3) Find the area of the surface generated by revolving the curve about the x -axis.

5. Given a polar curve $r = 1 + \sin \theta$, $0 \leq \theta \leq 2\pi$.

- (1) Find the tangent line to the polar curve at the point $(r, \theta) = (1, 0)$.
(2) Find the area of the region enclosed by the polar curve.
(3) Find the length of the polar curve.

6. Let $\mathbf{r}(t) = (4 \cos t)\mathbf{i} + (\sqrt{2} \sin t)\mathbf{j}$ be the position vector of a particle moving along a smooth curve from the initial point $A(4, 0)$ at $t = 0$.

- (1) Find the velocity (速度) and acceleration (加速度).
(2) Find the minimum speed (最小速率) of the particle during the period $t \in [0, 2\pi]$.
(3) Find the unit tangent vector of the particle at $t = 0$.

7. Let $\mathbf{r}(t) = (6 \sin 2t)\mathbf{i} + (6 \cos 2t)\mathbf{j} + (5t)\mathbf{k}$, $0 \leq t \leq \pi$, be a vector-valued function.

- (1) Find the derivative of $\mathbf{r}(t)$ at $t = 0$.
(2) Evaluate the definite integral $\int_0^{\pi} \mathbf{r}(t) dt$.
(3) Find the curvature (曲率) of the curve.
(4) Find the principle unit normal of the curve.