

1. (a) Find all the local maxima, local minima and saddle points of $f(x, y) = x^3 + 3xy + y^3$. (10 pts)
(b) Find the absolute maxima and minima of $g(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the triangular region bounded by the lines $x = 0$, $2 - y = 0$ and $2x - y = 0$. (10 pts)
2. Find the volume of the given solid region using the *cylindrical* or *spherical* coordinates.
(a) The solid bounded above by the plane $z = 4 - y$ and below by the circular region $R = \{(x, y) \in \mathbb{R}^2 \mid x^2 + (y - 1)^2 \leq 1\}$ in the xy -plane. (10 pts)
(b) The solid bounded below by the sphere $x^2 + y^2 + (z - 1)^2 = 1$ and above by the cone $z = \sqrt{x^2 + y^2}$. (10 pts)
3. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at the point $(u, v) = (1/2, 1)$ if $w = xy + yz + xz$, $x = u + v$, $y = u - v$ and $z = uv$. (7 pts)
4. Evaluate the following iterated integrals. (18 pts)
(a) $\int_0^{2/3} \int_y^{2-2y} (x+2y)e^{(y-x)} dx dy$ (b) $\int_0^{\pi/4} \int_y^{\pi/4} \frac{\sec^2 x}{x} dx dy$ (c) $\int_0^1 \int_x^{\sqrt{2-x^2}} (x+2y) dy dx$
5. Let $f(x, y, z) = 3e^x \cos(yz)$ and $P_0(0, 0, 0)$ be a point in its domain.
(a) Find the gradient of f at the point P_0 . (5 pts)
(b) Use (a) to find the directional derivative of f at the point P_0 in the direction of $\vec{v} = 2\vec{i} + \vec{j} - 2\vec{k}$. (5 pts)
(c) Find the direction in which f decreases most rapidly at the point P_0 . (5 pts)
6. Find a parametric equation for the line tangent to the curve of intersection of the surfaces $x + y^2 + 2z - 4 = 0$ and $x = 1$ at the point $(1, 1, 1)$. (10 pts)
7. Find the maximum and minimum values of $x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 - 4y = 0$. (10 pts)