

1. Evaluate the following iterated integrals. (5 pts each)

(a) $\int_1^e \int_1^{e^2} \int_1^{e^3} \frac{1}{xyz} dx dy dz$

(b) $\int_0^1 \int_{\sqrt[3]{y}}^1 \frac{2\pi \sin(\pi x^2)}{x^2} dx dy$

(c) $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} e^{x^2+y^2+2} dx dy$

(d) $\int_0^1 \int_{-y}^y 4(x+y)e^{x-y} dx dy$

2. Evaluate $\lim_{(x,y) \rightarrow (1,1)} \frac{xy^2 - 1}{y - 1}$ if it exists. If the limit does not exist, explain why. (10 pts)

3. Let $z = ye^{2xy}$ be a surface and $P(0, 2, 2)$ be a point in its domain. (5 pts each)

(a) Find an equation of the tangent plane at P .

(b) Find the normal line to the surface at P .

4. Find the derivative of $f(x, y, z) = xyz$ in the direction of the velocity vector of the helix $\mathbf{r}(t) = (\cos 3t)\mathbf{i} + (\sin 3t)\mathbf{j} + 3t\mathbf{k}$ at $t = \frac{\pi}{3}$. (10 pts)

5. (a) Find all the local maxima, local minima and saddle points of the function $f(x, y) = e^x(x^2 - y^2)$. (10 pts)

(b) Find the absolute maxima and minima of the function $f(x, y) = x^2 - xy + y^2 + 1$ on the closed triangular plate in the first quadrant bounded by the lines $x = 0$, $y = 4$ and $y = x$. (10 pts)

6. Find the average value of the function $f(x, y) = \sqrt{4 - x^2 - y^2}$ above the disk $x^2 + y^2 \leq 4$ in the xy -plane. (10 pts)

7. (a) Use cylindrical coordinate to find the volume of the solid bounded above by $z = 2 - x^2 - y^2$ and below by $z = x^2 + y^2$. (10 pts)

(b) Use spherical coordinate to find the volume of the solid inside $x^2 + y^2 + z^2 = 9$ outside $z = \sqrt{x^2 + y^2}$ and above the xy -plane. (10 pts)