

國立臺灣師範大學104學年度上學期微積分乙(一) 期中考

考試規範:

1. 禁止帶手機、計算機及電腦等電子產品進入考場。
2. 所有題目的解答都要有計算過程，無計算過程不給分。

1. Find the limits:

- (a) (5 points) $\lim_{x \rightarrow 9} \frac{9x - x^2}{3 - \sqrt{x}}$
- (b) (5 points) $\lim_{x \rightarrow \infty} \frac{5x^4 + 2x - 7}{2x^4 - 3x^3 - x + 6}$
- (c) (5 points) $\lim_{x \rightarrow \infty} e^{1/x} \cos\left(\frac{1}{x}\right)$
- (d) (5 points) $\lim_{x \rightarrow 0} \frac{\tan^2(3x)}{x \sin(5x)}$
- (e) (5 points) $\lim_{x \rightarrow 0^-} \left(x^2 - \frac{1}{x}\right)$

2. (10 points) Find the horizontal, vertical and slant asymptotes (水平、垂直和斜漸近線) of the graph of $f(x) = \frac{x^3 - 1}{2x^2 - 4}$

3. Find dy/dx :

- (a) (5 points) $y = 2^x \left(1 - \frac{4}{x+3}\right)$
- (b) (5 points) $y = \left(\ln \frac{3}{x}\right)^{x/3}$
- (c) (5 points) $y = e^{3x} \sin(\cos^2(3x))$
- (d) (5 points) $y = \sqrt{2 + \sqrt{2 + \sqrt{x}}}$
- (e) (5 points) $\ln(xy) - e^{x+y} + x^2y = 1$

4. $f(x) = 2x + \frac{1}{x}$

- (a) (5 points) Find all the critical points (臨界點) of $f(x)$.
- (b) (10 points) Find the interval(s) on which $f(x)$ is increasing or decreasing.
- (c) (5 points) Find the extreme values.

5. $f(x) = x^3 - 9x^2$

- (a) (5 points) Find all the points of inflection (反曲點) of $f(x)$.
- (b) (10 points) Find the interval(s) on which the curve $f(x)$ is concave up or down.
- (c) (5 points) Find the extreme values.

6. **Bonus:** choose only one question from the following questions.

- (a) (5 points) Show that $\lim_{x \rightarrow 3} (8 - x) = 5$, using the precise definition, ϵ, δ of the limit.
- (b) (5 points) Show that $e = \lim_{x \rightarrow 0} (1 + x)^{1/x}$