

1. (10 pts) Find the absolute extrema (絕對極值) of $f(x) = 5e^x - e^{2x}$ on the closed interval $[-1, 2]$.
2. (20 pts) Find the limit.

$$(a) \lim_{x \rightarrow \infty} \frac{5 - 2x^{3/2}}{3x^2 - 4} \quad (b) \lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x - 4}$$

$$(c) \lim_{\theta \rightarrow 0} \theta^2 \cos\left(\frac{1}{\theta}\right) \quad (d) \lim_{t \rightarrow 0} \frac{1 - e^{-t}}{e^t - 1}$$

3. For $f(x) = \frac{x^2 + 2}{x^2 - 2}$, answer the following questions.
 - (a) (5 pts) Determine the open intervals on which f is increasing (遞增) or decreasing (遞減).
 - (b) (5 pts) Find all relative extrema (相對極值) for f .
 - (c) (5 pts) Determine the open intervals on which the graph of f is concave upward (凹向上) or concave downward (凹向下).
 - (d) (5 pts) Find all points of inflection (反曲點) of the graph of f .
 - (e) (5 pts) Find all horizontal asymptotes (水平漸近線) for f .
 - (f) (5 pts) Find all vertical asymptotes (垂直漸近線) for f .
4. (5 pts) Find the constant a such that the function

$$f(x) = \begin{cases} \frac{\sin 2x}{3x}, & x < 0, \\ \frac{1}{6}a - 7x, & x \geq 0 \end{cases}$$

is everywhere continuous (處處連續).

5. (10 pts) Use **Intermediate Value Theorem** and **Rolle's Theorem** to prove that the equation $2x^5 + 7x - 1 = 0$ has *exactly one* real solution.
6. (5 pts) Find an equation of the tangent line (切線) to the graph of

$$x^2 + xy + y^2 = 4$$

at the point $(2, 0)$.

7. Consider a real-valued function (實值函數) defined by

$$g(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

- (a) (5 pts) Is g differentiable (可微分) at $x = 0$? If yes, find $g'(0)$.
(b) (5 pts) Find the derivative of g for all $x \neq 0$.
8. (10 pts) Find the differential dy of the given function.

(a) $y = \ln \sqrt{4 - x^2}$ (b) $y = \arctan(x - 2)$

9. (加分題, 5 pts) Use the ε - δ definition to show that

$$\lim_{x \rightarrow -2} (4x + 5) = -3.$$