

Midterm exam 19/Apr/2017

1. (5 pts) Determine the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{5/7}}.$$

2. (5 pts) Determine the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \left(\frac{n}{4n+1} \right)^n.$$

3. (10 pts) Determine the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \ln \left(1 + \frac{1}{n^2} \right).$$

4. (10 pts) For each positive real number p , determine the convergence or divergence of the series

$$\sum_{n=3}^{\infty} \frac{1}{n(\ln n)(\ln \ln n)^p}.$$

5. (5 pts) Determine the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n!}}.$$

6. (5 pts) Find

$$\sum_{n=1}^{\infty} \frac{1}{n^3 + 3n^2 + 2n}.$$

7. (5 pts) Find the radius of convergence of the following power series

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{(2n)!}.$$

8. (5 pts) Find the intervals of convergence of the following series

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}.$$

9. (10 pts) Use the definition of Taylor series to find the Taylor series, centered at $c = 0$, for the function $f(x) = \sin 3x$.
10. (10 pts) Find the power series, centered at $c = 0$, for the function $f(x) = \ln(x^2 + 1)$, and determine the interval of convergence.
11. (15 pts) Consider the function

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x - 5)^n}{n \cdot 5^n}$$

Find the intervals of convergence of (a) $f(x)$, (b) $f'(x)$, (c) $\int f(x)dx$.

12. (10 pts) Find the power series, centered at $c = -3$, for the function

$$f(x) = \frac{5}{2x - 3},$$

and determine the interval of convergence.

13. (5 pts) By eliminating the parameters, write the rectangular equation for the following parametric equations

$$x = t^2 + t, \quad y = t^2 - t.$$

14. (5 pts) Find the unit tangent vector to the curve at the specified value of the parameter.

$$\mathbf{r}(t) = 3t \mathbf{i} - \ln t \mathbf{j}, \quad t = e.$$

15. (5 pts) Find the principal unit normal vector to the curve at the specified value of the parameter.

$$\mathbf{r}(t) = 6 \cos t \mathbf{i} + 6 \sin t \mathbf{j} + \mathbf{k}, \quad t = \frac{3\pi}{4}.$$