

105 學年度微積分(乙)下學期期末考

1. Find the direction derivative of the function $h(x, y, z) = x \tan^{-1} yz$ at the point $P(4, 1, 1)$ in the direction $\vec{v} = \vec{i} + 2\vec{j} - \vec{k}$. (8 points)
2. Find the absolute extrema of the function $f(x, y) = 12 - 3x - 2y$ over the region R , where R is the triangular region in the xy -plane with vertices $(2, 0)$, $(0, 1)$, and $(1, 2)$. (10 points)

3. Convert the integral from rectangular coordinates to both cylindrical and spherical coordinates, and evaluate the simplest iterated integral. (10 points)

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^4 x \, dz \, dy \, dx.$$

4. Find both first partial derivatives of following function. (8 points)

$$f(x, y) = \int_x^y (2t+1) \, dt + \int_y^x (2t-1) \, dt.$$

5. Differentiate implicitly to find the first partial derivatives of z . (6 points)

$$x \ln y + y^2 z + z^2 = 8.$$

6. Find an equation of the tangent plane and find a set of symmetric equation for the normal line to the surface at the given point. (10 points)

$$x^2 + y^2 + z = 9, \quad (1, 2, 4).$$

7. Set up integrals for both orders of integration. (6 points)

$$\iint_R x e^y \, dA, \quad R: \text{triangle bounded by } y = 4 - x, \quad y = 0, \quad x = 0.$$

8. Find the area of the surface given by $z = f(x, y)$ over the region R . (10 points)

$$f(x, y) = 8 + 4x - 5y, \quad R = \{(x, y) : x^2 + y^2 \leq 1\}.$$

9. Find the average value of the function over the given solid. (10 points)

$$f(x, y, z) = x + y \quad \text{over the solid bounded by the sphere } x^2 + y^2 + z^2 = 3.$$

10. Use the indicated change of variable to evaluate the double integral

$$\iint_R 4(x^2 + y^2) dA, \text{ where } R \text{ is the region bounded by the lines } x + y = -1,$$

$x + y = 1$, $x - y = -1$, and $x - y = 1$. (10 points)

$$x = \frac{1}{2}(u + v), \quad y = \frac{1}{2}(u - v).$$

11. Find $h(x, y) = g(f(x, y))$ and the set on which h is continuous. (5 points)

$$g(t) = t + \ln t \quad \text{and} \quad f(x, y) = \frac{1 - xy}{1 + x^2 y^2}.$$

12. Evaluate the iterated integral by converting to polar coordinates. (7 points)

$$\int_0^4 \int_0^{\sqrt{16-y^2}} (x^2 + y^2) dx dy.$$