

微積分乙 Calculus II Final Exam

注意1:本次期末考滿分110分

注意2:答題必須要有過程, 只有答案不予計分

1. Find the limit, if it exists, or show that the limit does not exist.

(a) (5pts) $\lim_{(x,y) \rightarrow (0,0)} \left(\frac{x^2 - y^2}{x^2 + y^2} \right)^2$.

(b) (5pts) $\lim_{(x,y) \rightarrow (0,0)} \frac{e^{-x^2 - y^2} - 1}{x^2 + y^2}$ (Hint: you may use polar coordinates).

2. (5pts) Let $z = z(x, y)$ be a function of two variables defined by the following equation:

$$e^{xz} + xy = 0.$$

3. Find the first partial derivatives of z , i.e., $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(a) (5pts) Find the equation of the tangent plane of the surface: $xyz = 4$ at the point $(1, 2, 2)$.

(b) (5pts) Find the equation of the normal line of the surface: $\sin xy + \sin yz = 1$ at the point $(\pi, \frac{1}{6}, \pi)$.

4. (10pts) Let $f(x, y) = x^3 + 3xy + y^2$ defined in $\mathbb{R}^2$. Find the critical points of $f(x, y)$ and determine the relative extrema or saddle points of them.

5. (10pts) Find the extreme values of $f(x, y, z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y - 4z = 49$.

6. Evaluate the following integrals:

(a) (5pts) $\int_1^4 \int_1^{\sqrt{x}} 2ye^{-x} dy dx$.

(b) (5pts) $\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{x^2+y^2} dx dy$.

7. Set up the integrals for both orders of integration and use the more convenient order to evaluate the integral.

(a) (7pts) $\int_R \int \frac{y}{1+x^2} dA$, R : the region bounded by $y = 0$, $y = \sqrt{x}$, $x = 4$.

(b) (7pts) $\int_R \int \sin x^2 dA$, R : the region bounded by $0 \leq y \leq 1$, $y \leq x \leq 1$.

(c) (7pts) $\int_R \int \frac{1}{x(1+x^2)} dA$, R : the region bounded by $0 \leq y \leq 1$, $\sqrt{y} \leq x \leq 1$.

8. Find the surface area of the followings:

(a) (7pts) The paraboloid $z = 1 + x^2 + y^2$ that lies above the region $x^2 + y^2 \leq 1$.

(b) (7pts) The surface given by $\{(x, y, z) \in \mathbb{R}^3 \mid z = 24 - 3x - 2y, x \geq 0, y \geq 0, z \geq 0\}$.

9. Find the volume of the solid region Q :

(a) (5pts) Q is the region enclosed by the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $\{(r, \theta, z) \mid r = 2 \sin \theta, 0 \leq \theta \leq \pi, z \in \mathbb{R}\}$.

(b) (5pts) Q bounded above by $x^2 + y^2 + z^2 = 9$ and below by $z^2 = x^2 + y^2$.

10. (10pts) Using technique of change of variable for two variables to evaluate the following integral:

$$\int_R \int (x+y)^4 (x-y)^2 dy dx$$

where R is the quadrilateral with four vertices $(1, 0)$, $(0, 1)$, $(-1, 0)$, $(0, -1)$. (Hint: you may let $u = x + y$ and $v = x - y$.)