

1. (10 pts) Find an equation in spherical coordinates (球面坐標) for the surface represented by the rectangular equations (直角坐標方程式) $x^2 + y^2 = z^2$ and $z \geq 0$.
2. Let $\mathcal{C}$ be a plane curve parametrized by

$$x = \arcsin t, \quad y = \ln \sqrt{1 - t^2}, \quad 0 \leq t \leq \frac{1}{2}.$$

- (a) (10 pts) Find the arc length (弧長) of $\mathcal{C}$.
- (b) (10 pts) Find the concavity (凹性) of $\mathcal{C}$ when $t = 1/4$.
3. (15 pts) Determine the convergence (收斂) or divergence (發散) of the series and give your reasons.

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n}{n \sin(1/n)} \quad (b) \sum_{n=1}^{\infty} \frac{n^2}{e^{n^2}} \quad (c) \sum_{n=3}^{\infty} \frac{1}{\sqrt{n \ln n}}$$

4. Suppose that a particle moves along the space curve given by

$$\mathbf{r}(t) = (5 \cos t - \cos 5t)\mathbf{i} + (5 \sin t - \sin 5t)\mathbf{j} + \frac{1}{t}\mathbf{k}, \quad 0 < t \leq 2\pi,$$

where t denotes the time parameter. (時間參數)

- (a) (10 pts) What is the speed (速率) of the particle at time $t = \pi$?
- (b) (10 pts) Find the acceleration (加速度) vector of the particle at time $t = \pi/2$.
5. (10 pts) Find the area of the polar region (極坐標區域) enclosed by $r = 2 \sin(3\theta)$.
6. (10 pts) Evaluate $\mathbf{r}(\frac{\pi}{4}) \times \mathbf{r}'(\frac{\pi}{4})$ if $\mathbf{r}(t)$ is a vector-valued function satisfying the following equation with initial conditions (初始條件)

$$\mathbf{r}''(t) = -4 \cos t \mathbf{j} - 3 \sin t \mathbf{k}, \quad \mathbf{r}'(0) = 3\mathbf{k}, \quad \mathbf{r}(0) = 4\mathbf{j}.$$

7. (a) (5 pts) Find a power series (冪級數) for $f(x) = \frac{1}{3-x}$ centered at 2.
- (b) (5 pts) Find the Taylor series (泰勒級數) for $\ln(3-x)$ at 2 using the Part (a).
- (c) (5 pts) Find the interval of convergence (收斂區間) for the power series obtained in the Part (b).
8. (加分題, 5 pts) Find the sum of the series $\sum_{n=1}^{\infty} \frac{n \arctan n + \arctan n - n \arctan(n+1)}{n^2 + n}$.