

1. (10 pts) If $u = x^4y + y^2z^3$, where $x = rse^t$, $y = rs^2e^{-t}$, and $z = r^2s \sin t$, find the value of $\frac{\partial u}{\partial s}$ when $r = 2, s = 1, t = 0$.
2. (10 pts) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $yz + x \ln y = z^2$ (in terms of x, y, z (以 x, y, z 表示)).
3. Let $f(x, y, z) = xe^y + ye^z + ze^x$, $P(0, 0, 0)$ and $Q(5, 1, -2)$.
 - (a) (5 pts) Find the directional derivative (方向導數) of f at P in the direction from P to Q .
 - (b) (5 pts) In which direction does f increase most rapidly (增加最快) at P ?
 - (c) (5 pts) What is the maximum rate of change (變化率) of f at P ?
4. Let $S_1 : x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0$, $S_2 : x^2 + y^2 + z^2 = 11$, and $P(1, 1, 3)$.
 - (a) (5 pts) Find an equation of the tangent plane (切平面) to the surface S_1 at point P .
 - (b) (5 pts) Let C be the curve of intersection (相交的曲線) of the surfaces S_1 and S_2 . Find an equation of the tangent line (切線) to the curve C at point P .
5. (10 pts) Determine the relative extrema (相對極值) of $f(x, y) = 6xy - x^6 - y^6$.
6. (10 pts) Find the minimum distance from $P(1, 2, 0)$ to the surface $z = \sqrt{x^2 + y^2}$.
7. Evaluate the following integrals:
 - (a) (5 pts) $\iint_R e^{-x^2-y^2} dA$, where R is the region bounded by $x = \sqrt{4 - y^2}$ and the y -axis.
 - (b) (5 pts) $\int_0^1 \int_x^1 \sin(y^2) dy dx$.
 - (c) (5 pts) $\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$, where R is the trapezoidal (梯形的) region with vertices $(1, 0)$, $(2, 0)$, $(0, 2)$ and $(0, 1)$.
8. (10 pts) Find the area of surface given by $z = xy$ over the region $x^2 + y^2 \leq 16$.
9. (10 pts) Find the volume of the solid region Q bounded below by $z = \sqrt{x^2 + y^2}$ and above by $x^2 + y^2 + z^2 = z$.
10. (Bonus, 5pts) Evaluate the convergent improper integral $\int_0^\infty e^{-x^2} dx$.