

注意事項: 無計算過程或說明不給分, 總分 100 分。

1. (25 pts) Find the limit (極限).

$$\begin{array}{lll} \text{(a)} \quad \lim_{\theta \rightarrow 0^+} \operatorname{arccot}\left(\frac{1}{\theta}\right) \sin^2\left(\frac{1}{\theta}\right) & \text{(b)} \quad \lim_{x \rightarrow 1} \frac{\sqrt{2+x} - \sqrt{3}}{1-x} & \text{(c)} \quad \lim_{x \rightarrow -\infty} \frac{4 - 5x^{2/3}}{2x^{4/5} + 3} \\ \text{(d)} \quad \lim_{z \rightarrow 0} \frac{5 - 5e^{2z}}{1 - e^z} & \text{(e)} \quad \lim_{x \rightarrow 0} 6x \cot(3x) & \end{array}$$

2. Let $y = g(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$ for $x > 0$.

(a) (5 pts) Find the differential dy .

(b) (5 pts) Use the part (a) to find the approximate value (近似值) of $g(4.16)$.

3. (10 pts) Does the nonlinear equation (非線性方程式) $e^{-3x} - x = 0$ have a unique solution (唯一解) in the interval $[0, 1]$? Give your reasons.

4. For the real-valued function (實值函數) $f(x) = \frac{x^3}{x^2 - 1}$, answer the following questions.

(a) (5 pts) Determine the open intervals on which f is increasing (遞增) or decreasing (遞減).

(b) (5 pts) Find the relative extrema (相對極值) for f .

(c) (5 pts) Determine the open intervals on which the graph of f is concave upward (凹向上) or concave downward (凹向下).

(d) (5 pts) Find the points of inflection (反曲點) of f .

(e) (5 pts) Find the vertical asymptotes (垂直漸近線) for f .

(f) (5 pts) Find the slant asymptotes (斜漸近線) for f .

5. Consider the real-valued function f defined by

$$f(x) = \begin{cases} x \cdot \operatorname{arcsec}\left(\frac{1}{x}\right) & \text{if } -1 \leq x < 0 \text{ or } 0 < x \leq 1, \\ 0 & \text{if } x = 0. \end{cases}$$

(a) (5 pts) Is f continuous (連續) on the closed interval $[-1, 1]$? Give your reasons.

(b) (5 pts) Find the first derivative (一階導函數) of f for $-1 < x < 0$ or $0 < x < 1$.

(c) (5 pts) Is f differentiable (可微分) at $x = 0$? Give your reasons.

6. (10 pts) Use the Logarithmic Differentiation (對數微分法) to find an equation of the tangent line (切線方程式) to the graph of $y = \frac{(x+2)^2}{\sqrt{x^2+1}}$ at the point $(0, 4)$.