

注意事項：無計算過程或說明不給分。總分 100 分，加分題 5 分。

1. Find the following limits (if it exists). If the limit does not exist, explain why.

(a) (5 pts) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^2 + y^2}$ (b) (5 pts) $\lim_{(x,y) \rightarrow (0,0)} \frac{5xy^2}{x^2 + 2y^4}$

2. Let $f(x, y, z) = xye^z$, $P(2, 4, 0)$ and $Q(0, 0, 0)$.

- (a) (5 pts) Find the directional derivative of f at P in the direction from P to Q .
(b) (5 pts) Find the maximum rate of change of f at P .

3. Let $F: x^2 + y^2 + z^2 + 2x - 4y - 4z - 12 = 0$, $G: 4x^2 + y^2 + 16z^2 = 24$ and $P(1, -2, 1)$.

- (a) (5 pts) Find an equation of the tangent plane to the surface F at the point P .
(b) (5 pts) Find an equation of the tangent line to the curve of intersection of F and G at the point P .

4. (10 pts) Find all relative extrema of $f(x, y) = 2xy - \frac{1}{2}(x^4 + y^4) + 1$.

5. Evaluate the following integrals.

(a) (5 pts) $\int_0^4 \int_0^{\pi/2} \int_0^{1-x} x \cos y \, dz dy dx$

(b) (5 pts) $\int_0^2 \int_{x^2/2}^2 \sqrt{y} \cos y \, dy dx$

(c) (5 pts) $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \sin(x^2 + y^2) \, dy dx$

(d) (5 pts) $\iint_R \frac{xy}{1 + x^2 y^2} \, dA$, where R : region bounded by the graph of $xy = 1$, $xy = 4$, $x = 1$ and $x = 4$.

6. (10 pts) Find points on the surface $x^2 + 4y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $x + 4y - z = 0$.

7. (10 pts) Find any extrema of the function $f(x, y) = e^{-\frac{xy}{4}}$ under the constraint $x^2 + y^2 = 1$.

8. (10 pts) Find the area of surface given by $f(x, y) = \ln |\sec x|$ over the region $R = \{(x, y) : 0 \leq x \leq \frac{\pi}{4}, 0 \leq y \leq \tan x\}$.

9. (10 pts) Find the volume of the solid inside $x^2 + y^2 + z^2 = 9$ outside $z = \sqrt{3(x^2 + y^2)}$ and above the xy -plane.

10. (Bonus, 5 pts) Use $\varepsilon - \delta$ to show that $f(x, y) = \frac{x + y}{x^2 + 1}$ is continuous at $(0, 0)$.