

考試規範：

1. 本試卷共六大題 (共一頁)，配分請見各題，合計 100 分。
2. 禁止帶手機、計算機及電腦等電子用品進入考場。
3. 所有題目的解答都要有計算過程，無計算過程不給分。

1. (5 pts×5) Determine the convergence or divergence of the following series. (You have to show the reason!!)

(a) $\sum_{n=1}^{\infty} 3n \sin\left(\frac{1}{2n}\right)$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+2}$ (c) $\sum_{n=1}^{\infty} \frac{1}{(n+3)^{2/3}}$

(d) $\sum_{n=2}^{\infty} \frac{1}{\ln(n!)}$ (e) $\sum_{n=1}^{\infty} \frac{n!}{n^{n+8}}$

2. Let C be the polar curve given by $r = 1 + \cos \theta$, $-\pi \leq \theta \leq \pi$.

- (a) (10 pts) Sketch the polar curve C .
(b) (10 pts) Find the area enclosed by C .

3. (a) (10 pts) Use definition to find the Maclaurin series for e^{-2x} .
(b) (5 pts) Show that e^{-2x} equals to its Maclaurin series.

4. The parametric curve is given by $x = t + \sin t$, $y = 1 - \cos t$, $0 \leq t \leq 2\pi$.

- (a) (5 pts) Find the equation of tangent line when $t = \pi/2$.
(b) (10 pts) Find the arc length of the curve between given interval.

5. (a) (10 pts) Find a power series of the function $\ln \frac{1+x}{1-x}$ centered at 0.
(b) (5 pts) Find the interval of convergence of the power series in (a).

6. (a) (5 pts) Find an equation in "Cylindrical" coordinates for the surface given by the Rectangular coordinates $\frac{x^2}{4} + \frac{y^2}{9} = 1$.
(b) (5 pts) Find an equation in "Rectangular" coordinates for the surface given by the Spherical coordinates $\rho = \csc \phi$.