

1. 本試卷共八大題 (共一頁), 配分請見各題, 合計 100 分。
2. 禁止帶手機、計算機及電腦等電子用品進入考場。
3. 所有題目的解答都要有計算過程, 無計算過程不給分。

1. (10 points) Find the following limits(if it exists). If the limit does not exist, explain why.

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - y^4}{x^2 + y^2}$ (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - y^2}{x^4 + y^2}$

2. Let function $f(x, y, z) = x^2 + y^2 + z^2$, point $P(1, 1, 1)$ and vector $\mathbf{v} = \mathbf{i} - \mathbf{j} + \mathbf{k} = (1, -1, 1)$.

- (a) (6 points) Find the directional derivative of f at P in the direction $\mathbf{v}$.
- (b) (4 points) Find the direction of maximum increase of f at the point P . What is this rate of increase?

3. Let surfaces $S_1 : x^2 + 2y^2 + 3z^2 = 6$, $S_2 : xyz = 1$ and point $P(1, 1, 1)$.

- (a) (6 points) Find an equation of the tangent plane to the surface S_1 at the point P .
- (b) (6 points) Find an equation of the tangent line to the curve of intersection of surfaces S_1 and S_2 at the point P .

4. (10 points) Find all relative extrema and saddle points of $f(x, y) = x^3 - y^3 - 2xy + 6$.

5. (10 points) Find the minimum distance from the surface $x^2 - y^2 - z^2 = 1$ to the origin.

6. (28 points) Evaluate the following integrals.

(a) $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy$

(b) $\int_0^2 \int_0^{\sqrt{4-x^2}} \sin(x^2 + y^2) dy dx$

(c) $\iint_R \sin(x - y) \cos^2(x + 2y) dA$ where R is the parallelogram on the xy plane bounded by $x - y = 0$, $x - y = \pi$, $x + 2y = 0$ and $x + 2y = \frac{1}{2}\pi$.

(d) $\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} (x^2 + y^2 + z^2) dz dy dx$

7. (10 points) Find the area of the surface $z = 6 - x^2 - y^2$ that lies above the region $x^2 + y^2 \leq 4$.

8. (10 points) Find the volume of the solid in the first octant bounded by the coordinate planes, the plane $y + z = 2$, and the cylinder $x = 4 - y^2$.