

注意事項：無計算過程或說明不給分，總分 100 分。

1. (7 pts) Use the differential to approximate (近似) the value of $\sqrt[3]{63.952}$.
2. Let $f(x) = x \cos(1/x)$ for $x \neq 0$.
 - (a) (5 pts) Show that f has a removable discontinuity (可移除不連續點) at $x = 0$.
 - (b) (5 pts) Is f differentiable (可微分) at $x = 0$ when $f(0)$ is defined as in the part (a)? Give your reasons.
3. Let $g(x) = 5e^x - e^{2x} + 1$ for all $x \in \mathbb{R}$.
 - (a) (5 pts) Use the **Second Derivative Test** to find the relative extrema (相對極值) of g .
 - (b) (10 pts) Prove that the nonlinear equation $g(x) = 0$ has *exactly one* root (零根) in the closed interval $[0, \ln 10]$.
4. (28 pts) Find the limit (極限).
 - (a) $\lim_{x \rightarrow -\infty} (2x + \sqrt{4x^2 + 1})$
 - (b) $\lim_{x \rightarrow 0^+} \frac{\sqrt{x+1} - 1}{\sqrt{x^2+1} - \sqrt{x+1}}$
 - (c) $\lim_{\theta \rightarrow 0} \theta^2 \cot(7\theta) \csc(4\theta)$
 - (d) $\lim_{t \rightarrow 0} \ln \sqrt{(1+t)^{\frac{1}{3t}}}$

5. For $f(x) = \frac{2x^3 - 6x^2 + 4x}{x^2 - 1}$, answer the following questions.

- (a) (5 pts) Describe the domain (定義域) of f .
 - (b) (5 pts) Find the critical numbers (臨界數) of f .
 - (c) (5 pts) Determine the open intervals on which f is increasing (遞增) or decreasing (遞減).
 - (d) (5 pts) Determine the open intervals on which the graph of f is concave upward (凹向上) or concave downward (凹向下).
 - (e) (5 pts) Find all vertical asymptotes (垂直漸近線) for f .
 - (f) (5 pts) Find the slant asymptote (斜漸近線) for f .
6. (10 pts) Find an equation of the tangent line (切線) to the graph of

$$3e^{x+y} + \ln(x/y^2) - \arcsin(x+y) = 3$$

at the point $(1, -1)$.