

考試規範：

1. 本試卷共六大題 (共一頁), 配分請見各題, 合計 100 分。
2. 禁止帶手機、計算機及電腦等電子用品進入考場。
3. 所有題目的解答都要有計算過程, 無計算過程不給分。

1. (a) (10 pts) Find a power series of the function $\frac{4}{3x+2}$ centered at 3.
(b) (5 pts) Find the interval of convergence of the power series in (a).
2. The parametric curve is given by $x = e^{-t} \cos t$, $y = e^{-t} \sin t$, $0 \leq t \leq \pi/2$.
(a) (7 pts) Find the equation of the tangent line when $t = 0$.
(b) (8 pts) Find the arc length of the curve between given interval.
3. Let C be the polar curve given by $r = 2\sqrt{\cos(2\theta)}$, $-\pi/4 \leq \theta \leq \pi/4$.
(a) (10 pts) Sketch the polar curve C .
(b) (10 pts) Find the area enclosed by C .
4. (5 pts) Find the 3rd (third) Taylor polynomial for the function $\ln x$ centered at 2.
5. (a) (6 pts) Find an equation in "Spherical" coordinates for the surface given by the Rectangular coordinates $x^2 + y^2 = 4y$.
(b) (6 pts) Find an equation in "Rectangular" coordinates for the surface given by the Cylindrical coordinates $r^2 \cos(2\theta) - z^2 + 1 = 0$.
(c) (8 pts) Find an equation in "Rectangular" coordinates for the surface given by the Spherical coordinates $\phi = \pi/4$.
6. (5 pts×6) Determine the convergence or divergence of the following series. (You have to show the reason!!)

(a)
$$\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^2 - 3}$$

(b)
$$\sum_{n=1}^{\infty} \left(1 + \frac{\pi}{n}\right)^n$$

(c)
$$\sum_{n=1}^{\infty} \frac{3n^2}{n^6 + 2n^3 + 1}$$

(d)
$$\sum_{n=1}^{\infty} \frac{n2^n}{4n^3 + 1}$$

(e)
$$\sum_{n=0}^{\infty} \frac{(-1)^n 2^{4n}}{(2n+1)!}$$