

1. 本試卷共十大題 (共一頁), 配分請見各題, 合計 100 分。
2. 禁止翻閱書本、筆記和查詢相關資料。
3. 禁止使用手機、計算機及電腦等電子用品計算題目和獲得答案。
4. 所有題目的解答都要有計算過程, 無計算過程不給分。

1. (7 points) Find an equation of the tangent plane to the surface $\sin xy = 2 \cos yz$ at the point $(\pi/2, 1, \pi/3)$.
2. (7 points) Find the directional derivative of the function $f(x, y, z) = \ln \sqrt{x^2 + y^2} + e^{yz}$ at the point $P(1, 0, 2)$ in the direction toward the origin.

3. (5 points) Let $f(x, y) = \begin{cases} \frac{3xy}{\sqrt{x^2 + y^2}} & (x, y) \neq (0, 0); \\ c & (x, y) = (0, 0). \end{cases}$

Find a number c such that $f(x, y)$ is continuous at $(0, 0)$.

4. Let $f(x, y) = \begin{cases} \frac{5x^2y}{x^4 + y^2} & (x, y) \neq (0, 0); \\ 0 & (x, y) = (0, 0). \end{cases}$

- (a) (5 points) Find $f_x(0, 0)$.
 - (b) (5 points) Find $f_x(x, y)$ for $(x, y) \neq (0, 0)$.
 - (c) (5 points) Does $\lim_{(x, y) \rightarrow (0, 0)} f(x, y)$ exist? If yes, find the value. If not, give the reason.
 - (d) (4 points) Is f differentiable at $(0, 0)$? Give your reason.
5. (8 points) Find all local maxima, local minima and saddle points of

$$f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2.$$

6. (8 points) Minimize xy^2 on the unit circle $x^2 + y^2 = 1$.
7. (16 points) Evaluate the following integrals.

(a) $\int_0^1 \int_x^1 x \sqrt{1 + y^3} \, dy \, dx$

(b) $\iint_R (x - y) \cos[\pi(x - y)] \, dA$ where R is the parallelogram on the xy plane bounded by $x + y = 0$, $x + y = 1$, $x - y = 0$ and $x - y = 2$.

8. (10 points) Find the volume of the solid bounded by $z = 5 - \sqrt{x^2 + y^2}$ above the plane region $R : x^2 + y^2 \leq 2y$.
9. (10 points) Use triple integration to find the volume of the solid bounded above by the parabolic cylinder $z = 4 - y^2$ and bounded below by $z = 2x^2 + y^2$.
10. (10 points) Convert the integral $\int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{\sqrt{16-x^2-y^2}} \sqrt{x^2 + y^2 + z^2} \, dz \, dy \, dx$ from rectangular coordinates to both cylindrical and spherical coordinates, and evaluate the simpler iterated integrals.