

考試規範：

1. 本試卷共九大題(共一頁)，配分請見各題，含加分題合計 105 分。
2. 禁止翻閱書本、筆記和查詢相關資料。
3. 禁止使用手機、計算機及電腦等電子用品計算題目和獲得答案。
4. 所有題目的解答都要有計算過程，無計算過程不給分。

1. Given the function $f(x) = x^4 + 4x^2 + 5$, $x > 0$. (15 pts)

- (a) Show that f is one to one.
- (b) Find the inverse function f^{-1} .
- (c) Find the derivative $(f^{-1})'(10)$.

2. Find the limit if it exists. Notice that L'Hôpital's Rule is forbidden. (20 pts)

- (a) $\lim_{x \rightarrow 2} \frac{\sqrt{x+7} - 3}{x-2}$ (b) $\lim_{x \rightarrow 0} \frac{\tan(5x)}{\sin(3x)}$ (c) $\lim_{x \rightarrow -\infty} (x + \sqrt{x^2 + 3})$
- (d) $\lim_{x \rightarrow \infty} \frac{\sqrt[3]{8x^6 + 2x^3 + 1}}{\sqrt{4x^4 + 3x + 1}}$

3. (a) Write the definition of the derivative $f'(x_0)$ of a function f at a point x_0 . (2 pts)
- (b) Show that $f(x) = |x|$ is continuous but not differentiable at $x_0 = 0$. (8 pts)

4. Finding the derivative $\frac{dy}{dx}$. (15 pts)

- (a) $y = (\sin x)^{\cos x}$ (b) $y = \ln \left(\frac{e^x - 1}{e^x + 1} \right)$ (c) $y = \arctan \frac{x}{2} - \frac{1}{2(x^2 + 4)}$.

5. Given $x^3 + ye^x + y^3 = 2$. Find the tangent line at point $P = (0, 1)$. (6 pts)

6. Use differential to approximate $\sqrt[4]{626}$. (6 pts)

7. Prove that the equation $2x - 2 - \cos x = 0$ has exactly one real solution. (6 pts)

8. Given a function $f(x) = x^4 - 2x^2$ on the interval $[-2, 3]$.

- (a) Find open intervals where f is increasing or decreasing. (6 pts)
- (b) Find the relative extrema of f . (10 pts)
- (c) Determine open intervals where f is concave upward or downward. (6 pts)

9. (加分題)

- (a) State the definition of limit $\lim_{x \rightarrow x_0} f(x) = l$. (2 pts)
- (b) Prove that $\lim_{x \rightarrow 2} (3x + 1) = 7$ by the definition of limit. (3 pts)