

1. Determine the convergence or divergence of the following series, and if it is convergence, determine it to be conditionally converged or absolutely converged. (You have to give your reason)

(1). $\sum_{n=1}^{\infty} \left(\frac{2n}{5n+2}\right)^n$; (2). $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}-\sqrt{n}}$; (3). $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)}$; (4). $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$.

(5 points for each problem)

2. Find the value of $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)(n+3)}$. (6 points)

3. By eliminating the parameters, write the rectangular equation for the following parametric equations: $x = t^2 + t$, $y = t^2 - t$. (6 points)

4. By using the definition of the Maclaurin series goes to find the Maclaurin series for the following functions. (8 points for each problem)

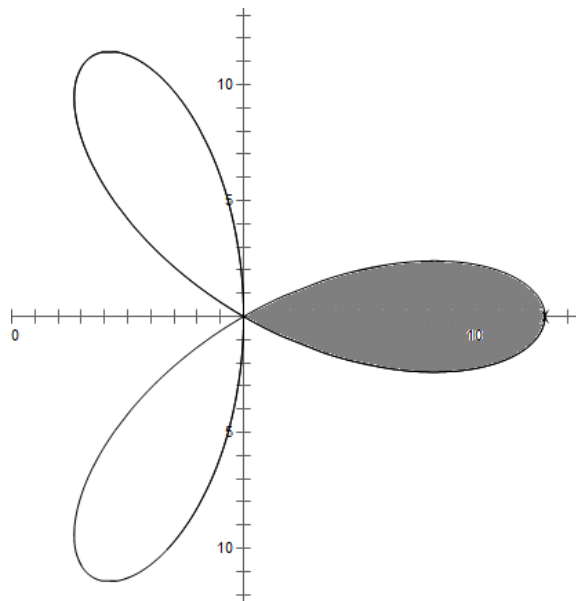
(1). $f(x) = \sin 2x$; (2). $g(x) = \ln(x^2 + 3)$.

5. Let $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \cdot 3^n} (x-3)^n$.

(1) Represent $f'(x)$ and $\int f(x)dx$ in terms of power series. (10 points)

(2) Find the interval of convergence of $f(x)$, $f'(x)$ and $\int f(x)dx$. (15 points)

6. Find the area of one petal of $r = 13 \cos 3\theta$. (7 points)



7. Find a set of parametric equations for the rectangular equation $y = 2x - 3$ that satisfies the condition $t = 0$ at the point $(2, 1)$. (7 points)

8. Find the open interval on which the curve $\vec{r}(\theta) = (1 + \sin \theta)\vec{i} + (1 - 2 \cos \theta)\vec{j}$ is smooth. (6 points)
9. Suppose $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^2$. Find the component form of vector $\vec{u} + \vec{v}$ given by $\|\vec{u}\| = 2$, $\|\vec{v}\| = 3$ and the angles that $\vec{u}$ and $\vec{v}$ with positive x -axis are $\theta_{\vec{u}} = 0^\circ$ and $\theta_{\vec{v}} = 45^\circ$. (7 points)
10. (Bonus) Suppose the sequences $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ satisfying $0 \leq a_n \leq c_n \leq b_n$ for all $n \in \mathbb{N}$, and L be a positive real number. Show that if $\sum_{n=1}^{\infty} a_n = L = \sum_{n=1}^{\infty} b_n$, then $\sum_{n=1}^{\infty} c_n = L$. (5 points)
11. (Bonus) Find the value of $\lim_{n \rightarrow \infty} \frac{\ln(n!)}{n \ln(n)}$. (5 points)