

1. 本試卷共九大題(共一頁), 配分請見各題, 合計 100 分。
2. 禁止翻閱書本、筆記和查詢相關資料。
3. 禁止使用手機、計算機及電腦等電子用品。
4. 所有題目的解答都要有計算過程, 無計算過程不給分。

1. (3 pts) Evaluate $\sin(2 \sin^{-1}(-\frac{3}{5}))$. (note : $\sin^{-1} x = \arcsin x$)

2. (18 pts) Find the limit if it exists.

(a) $\lim_{x \rightarrow 0} \frac{\sin 3x - 3x + x^2}{\sin x \sin 2x}$ (b) $\lim_{x \rightarrow \infty} \frac{3x}{5x + 2 \sin x}$ (c) $\lim_{x \rightarrow 0} \left(\frac{1}{x\sqrt{1+x}} - \frac{1}{x} \right)$

3. (18 pts) Find the derivatives $\frac{dy}{dx}$.

(a) $y = \sec^{-1}(\sqrt{x^2 + 4}) + \cot(3^x)$, (note : $\sec^{-1} x = \operatorname{arcsec} x$)

(b) $y = \ln(e^{2x} + e^{\sin 3x})$

(c) $y = (\sin x)^{\sqrt{x}}$, $x > 0$

4. (7 pts) Given $x \tan^{-1}(x^2) = e^y$. Find the tangent line at point $(1, \ln \frac{\pi}{4})$.

(note : $\tan^{-1} x = \arctan x$)

5. Let $f(x) = x - \pi + \cos x$.

(a) (3 pts) Show that f has an inverse on $(-2\pi, 2\pi)$.

(b) (5 pts) Find $(f^{-1})'(-1)$.

6. (8 pts) Prove that the equation $x + e^{x^3} = 0$ has exactly one real root.

7. (8 pts) Find all critical points and all local extrema (relative extrema) of the function

$$f(x) = x^{1/3}(x^2 - 7),$$

verify your answer by the first or the second derivative test.

8. (18 pts) Let $f(x) = \frac{2x^2 + 6}{x + 1}$.

(a) Find all asymptotes(漸進線).

(b) Determine the open intervals on which f is increasing(遞增) or decreasing(遞減).

(c) Determine the open intervals on which f is concave upward(凹向上) or concave downward(凹向下).

9. (12 pts) Let $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}), & x \neq 0; \\ 0, & x = 0. \end{cases}$

(a) Use the definition of continuity to show that f is continuous at $x = 0$.

(b) Is f differentiable at $x = 0$? Explain it.