

國立臺灣師範大學 111 學年度第 1 學期

微積分乙(一) Calculus B(I) 期末考 Final Exam

註：

- 統一考試日期時間：12/21(三) 18:40 ~ 20:40
- 無作答過程或理由以 0 分採計。
- 成績評量須依照全校微乙課程共同評分標準與規範。
- 總分 100 分，另有加分題 (Bonus) 8 分。試題共 5 大題。

1. (60 pts) Evaluate the integrals.

(1) (8 pts)

$$\int \frac{\ln(\ln x)}{x \ln x} dx$$

(2) (8 pts)

$$\int_0^1 x \ln x dx$$

(3) (8 pts)

$$\int \frac{x^3}{x^2 + 2x + 1} dx$$

(4) (8 pts)

$$\int \frac{x}{\sqrt{1+x^4}} dx$$

(5) (7 pts)

$$\int \sqrt{\sin x} \cos^3 x dx$$

(6) (7 pts)

$$\int \frac{x e^{2x}}{(2x+1)^2} dx$$

(7) (7 pts)

$$\int \frac{\sec^2 \theta}{\tan^3 \theta - \tan^2 \theta} d\theta$$

(8) (7 pts)

$$\int_{-\infty}^1 \frac{9r^2}{\sqrt{1-r^3}} dr$$

2. (8 pts) Find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$ and the line $y = x + 2$ about the line $x = 2$.

3. (10 pts) Solve the initial value problem for y as a function of x

$$\sqrt{x^2 - 9} \frac{dy}{dx} = 1, x > 3, y(5) = \ln 3.$$

4. (10 pts) Consider the well-known **standard logistic function**(標準型邏輯斯諦函數)

$$f(x) = \frac{1}{1 + e^{-x}}.$$

A typical(典型的) application is a model(模型) of population growth(人口成長), and the logistic functions are widely(廣泛地) used in probability(機率), statistics(統計), machine learning(機器學習), chemistry(化學), social science(社會科學) and so on.

(1) (6 pts) Find all antiderivatives of $f(x)$.

(2) (4 pts) Show that $f(x)$ is a solution of the first-order non-linear ordinary differential equation(一階非線性常微分方程)

$$\frac{dy}{dx} = y(1 - y)$$

with the initial value condition $y(0) = \frac{1}{2}$.

5. (20 pts) Consider the **error function**(誤差函數)

$$F(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

in areas(領域) of probability(機率) and statistics(統計).

(1) (2 pts) Is $F(x)$ continuous?

(2) (5 pts) Does the graph of $F(x)$ have horizontal tangent lines?

(3) (5 pts) Does the graph of $F(x)$ have inflection points(反曲點)?

(4) (Bonus) (8 pts) Does the graph of $F(x)$ have horizontal asymptotes(水平漸近線)? (Hint: The direct comparison test below will be useful.)

The Direct Comparison Test(直接比較檢測法): Let f and g be continuous on $[a, \infty)$ with $0 \leq f(x) \leq g(x)$ for all $x \geq a$. Then

◆ if $\int_a^\infty g(x)dx$ converges, then $\int_a^\infty f(x)dx$ also converges;

◆ if $\int_a^\infty f(x)dx$ diverges, then $\int_a^\infty g(x)dx$ also diverges.