

國立台灣師範大學 111 學年度下學期微積分乙 (二)

考試規範：無計算過程或說明不給分。

總分:100 分

1. (6 pts×6) Determine the convergence or divergence of the following series.

(1) $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+2} - \sqrt{n})$

(2) $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$

(3) $\sum_{n=1}^{\infty} \frac{n+3^n}{n3^n}$

(4) $\sum_{n=1}^{\infty} \frac{2 \tan^{-1}(n)}{1+n^2}$

(5) $\sum_{n=1}^{\infty} \frac{\ln n}{n^{5/4}}$

(6) $\sum_{n=1}^{\infty} \frac{2^{n^2}}{n^{2^n}}$

2. (8 pts) Find the value of $\sum_{n=1}^{\infty} \frac{10n+10}{(n^2+2n)^2} = ?$

3. Let $f(x) = \cos\left(\frac{\pi x}{3}\right)$

(1) (10 pts) Use definition to find the Maclaurin series of $f(x)$.

(2) (6 pts) Show that $f(x)$ equals to this Maclaurin series.

4. (10 pts) Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(n!)^2 x^n}{2^n (2n)!}$.

5. Let the curve $(x(t), y(t))$ satisfy the equations

$$x^3 + 2t^2 = 35, \quad 2y^3 - 3t^2 = 20.$$

(1) (5 pts) Compute the slope of this curve at $t = 2$.

(2) (5 pts) Compute $\frac{d^2 y}{dx^2}$ at $t = 2$.

6. Let C be the polar curve given by $r = 2 + 2 \sin \theta$, $0 \leq \theta \leq 2\pi$.

(1) (5 pts) Sketch the polar curve C .

(2) (7 pts) Find the area enclosed by C .

(3) (8 pts) Find the length of C .