

考試規範：

1. 本試卷共六大題 (共一頁)，配分請見各題，合計 110 分。
2. 禁止帶手機、計算機及電腦等電子用品進入考場。
3. 所有題目的解答都要有計算過程，無計算過程不給分。

1. (15 points) Find the following limits. If the limit does not exist, explain why.

(a) $\lim_{x \rightarrow 0} \lim_{y \rightarrow 0} \frac{x^4 - y^6}{x^4 + y^6}$. (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - y^6}{x^4 + y^6}$. (c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^6 - y^6}{x^2 - y^2}$.

2. Let surfaces $S_1 : e^{y^2 - x^2} - z = 0$ and $S_2 : y = 1$. $P = (-1, 1, 1)$ is a point on S_1 and S_2 .

- (a) (6 points) Find an equation for the tangent plane to S_1 at P .
 - (b) (6 points) Find the normal line of S_1 at P .
 - (c) (8 points) Find the tangent line to the curve where S_1 and S_2 meet at P .
3. (a) (10 points) Find all local maxima, local minima and saddle points (if any exists) of

$$f(x, y) = x^2y + x^2 - 2y.$$

(b) (15 points) Determine the maximum and minimum points of $f(x, y) = x^2 + y^2 + \frac{3}{2}x + 1$ on the set $G = \{(x, y) | 4x^2 + y^2 = 1\}$. (不限高中或大學做法, Hint: G 是一橢圓)

4. Evaluate the following integrals.

- (a) (5 points) $\int_1^4 \int_1^e \frac{\ln x}{xy} dx dy$.
- (b) (5 points) $\int_0^\pi \int_0^\pi \int_0^{2 \sin \phi} \rho^2 \sin \phi d\rho d\phi d\theta$.
- (c) (5 points) $\int_0^\pi \int_x^\pi \frac{\sin y}{y} dy dx$.
- (d) (5 points) $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dx dy$.

5. (15 points) Use $u = x + 2y, v = x - y$ to evaluate the integral $\int_0^{2/3} \int_y^{2-2y} (x + 2y)e^{(y-x)} dx dy$ by writing it as an integral over a region in uv plane.

6. (15 points) Find the volume of the solid bounded below by the sphere $\rho = 2 \cos \phi$ and above by the cone $z = \sqrt{x^2 + y^2}$.