

國立臺灣師範大學 112 學年度上學期微積分乙(一)期中考

考試規範：

1. 本試卷共八大題（共二頁），配分請見各題，含加分題合計 105 分。
 2. 禁止翻閱書本、筆記和查詢相關資料。
 3. 禁止使用手機、計算機及電腦等電子用品計算題目和獲得答案。
 4. 所有題目的解答都要有計算過程，無計算過程不給分。
1. Find the limit if it exists. Note that L'Hopital's Rule is forbidden.

(a)(5 pts) $\lim_{x \rightarrow 0} \frac{x + x \sin 4x}{\sin 2x \cdot \cos 3x}$ (b)(5 pts) $\lim_{x \rightarrow 0} \sqrt[3]{x} \sin\left(\frac{1}{x}\right)$

(c)(5 pts) $\lim_{x \rightarrow 5} \frac{\frac{x}{x-4} - 5}{x-5}$ (d)(5 pts) $\lim_{x \rightarrow \left(\frac{-4}{3}\right)} \sin\left(2 \tan^{-1}(x)\right)$

2. Find the derivatives $\frac{dy}{dx}$.

(a)(5 pts) $y = (3x^2 - 2x + 1)e^{x^2}$ (b)(5 pts) $y = \sin(\cos(2x + 3))$

(c)(5 pts) $y = \frac{\sqrt{(x^2 + 1)(x - 1)}}{\sqrt[3]{x^2} \cdot \sqrt[4]{(x + 1)^3} \cdot \sqrt[5]{(x + 2)^4}}$ (Hint: Use logarithmic differentiation.)

3.(10 pts) Find an equation of tangent line to the graph of $\ln(xy) = \sin(x - y)$ at the point $(1, 1)$.

4. Given a function $f(x) = x^3 - 3x + 2$, $x > 1$.

(a)(3 pts) Show that f is a one-to-one function on the interval $(1, \infty)$.

(b)(2 pts) Let f^{-1} denote the inverse function of f , find $f^{-1}(4)$.

(c)(5 pts) Find the derivative $(f^{-1})'(4)$.

5.(10 pts) Show that the function $g(t) = \sqrt{t} + \sqrt{t+2} - 4$ has exactly one zero in the interval $(0, \infty)$.

(請翻頁續答)

6. For the function $f(x) = \frac{x^3 + 3x^2 + 3x + 1}{x^2 - 2x - 3}$, answer the following questions.
- (a) (5 pts) Find the vertical asymptotes(鉛直漸近線) for f .
 - (b) (5 pts) Find the oblique asymptotes(斜漸近線) for f .
 - (c) (5 pts) Determine the open intervals on which f is increasing(遞增) or decreasing(遞減).
 - (d) (5 pts) Find the relative extrema(相對極值) for f .
 - (e) (5 pts) Determine the open intervals on which the graph of f is concave upward(凹向上) or concave downward(凹向下).
7. (10 pts) Let $f(x) = x \cos x$, find $f^{(112)}(x)$.
8. (Bonus, 5 pts) Use the $\varepsilon - \delta$ definition to show that $\lim_{x \rightarrow 2} (7 - 3x) = 1$.

(考題結束)