

考試規範：

1. 本試卷共六大題 (共一頁)，配分請見各題，合計 100 分。
2. 禁止帶手機、計算機及電腦等電子用品進入考場。
3. 所有題目的解答都要有計算過程，無計算過程不給分。

1. (a) (6 pts) Find an equation in Cartesian coordinates of the polar equation $r = 4 \tan \theta \sec \theta$.
(b) (6 pts) Find an equation in polar coordinates of the Cartesian equation $(x+4)^2 + y^2 = 16$.
2. Let C be the polar curve given by $r = 1 + \cos \theta$.
(a) (6 pts) Find the derivative at the point where $\theta = \frac{\pi}{6}$.
(b) (8 pts) Find the area enclosed by C .
3. Given a power series $\sum_{n=2}^{\infty} \frac{(2x-1)^n}{n \ln n}$.
(a) (6 pts) Find the radius of convergence of the series.
(b) (6 pts) Find the value of x where the series converges absolutely.
(c) (6 pts) Find the value of x where the series converges conditionally.
4. Suppose the acceleration(加速度向量) of a paper plane is $\mathbf{a}(t) = (\sin t)\mathbf{i} - (\cos t)\mathbf{j} + (4 \sin t \cos t)\mathbf{k}$, initially (at time $t = 0$) it departed from point $(1, 0, -1)$ with velocity(速度向量) $\mathbf{v}(0) = \mathbf{i}$.
(a) (6 pts) Find the velocity vector at π seconds.
(b) (6 pts) Find the position vector(位置向量) as a function of t .
5. (8 pts) Find the length of the parametric curve $x = 4 \cos t + 4t \sin t$, $y = 4 \sin t - 4t \cos t$, $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.
6. (6 pts×6) Determine the convergence or divergence of the following series. (You have to show the reason!!)

(a) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$

(b) $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^3}$

(c) $\sum_{n=1}^{\infty} \sqrt{n} \sin \frac{1}{\sqrt{n}}$

(d) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(e) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$

(f) $\sum_{n=1}^{\infty} \frac{1}{\pi^{\ln n}}$