

國立臺灣師範大學 113 學年度上學期微積分乙(一)期末考

考試規範：

1. 本試卷共八大題（共二頁），配分請見各題，含加分題合計 105 分。
2. 禁止翻閱書本、筆記和查詢相關資料。
3. 禁止使用手機、計算機及電腦等電子用品計算題目和獲得答案。
4. 所有題目的解答都要有計算過程，無計算過程不給分。

1. Find the limit if it exists.

(1)(6 pts) $\lim_{x \rightarrow 0} \frac{x^2}{\ln(\cos 2x)}.$

(2)(6 pts) $\lim_{x \rightarrow 0^+} (e^{2x} + 2x)^{\frac{2}{x}}.$

(3)(6 pts) $\lim_{x \rightarrow 0^+} \frac{\int_{\sqrt{x}}^0 \sin(t^2) dt}{\sqrt{x^3}}.$

2. Evaluate the integrals.

(1)(8 pts) $\int \frac{\cos^5 x}{\sin^2 x} dx$

(2)(8 pts) $\int \frac{x^2}{\sqrt{49-x^2}} dx$

(3)(8 pts) $\int \frac{2x^3+4}{(x-1)^2(x^2+1)} dx$

(4)(8 pts) $\int \frac{\tan x (\sec^2 x + 4 \sec x)}{\sec^2 x + 5 \sec x - 6} dx$

(5)(8 pts) $\int_{-1}^1 \left(\sqrt{x^{10}} + x^2 \sec x \tan x + \frac{1}{1+x^2} \right) dx$

3.(10 pts) Find the volume of solid generated by revolving the region bounded by the semicircle

$y = \sqrt{4-x^2}$ and the x -axis about the line $y = -1$.

4. Let D be the plane region enclosed by $y = -\sqrt{x}$, $x = y$, $y = -2$.

(1) (2 pts) Sketch the region D .

(2) (10 pts) Find the volume of the solid generated by revolving the region D about the x -axis.

(請翻頁續答)

5.(10 pts) Find the area of the surface generated by revolving the curve $x = 2\sqrt{8-y}$, $0 \leq y \leq 5$, about y -axis.

6.(a) (5 pts) Evaluate $\int_0^1 \ln x \, dx$. (Note: this integral is an improper integral(瑕積分).)

(b) (5 pts) Let a be a real number and $a > 1$. If a satisfies the equation $\int_0^a \ln x \, dx = 0$, find a .

7.(Bonus, 5 pts) Evaluate $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{7 \cdot 2^7 \cdot k^6}{n^7} - \frac{4 \cdot 2^4 \cdot k^3}{n^4} + \frac{2}{n} \right)$.

(Hint: The question is about a limit of some Riemann sum(黎曼和). Find the definite integral that the limit represents and evaluate it.)

(考題結束)