

1 (10 points) Let

$$f(x, y, z) = x \arctan\left(\frac{y}{z}\right).$$

- Find the directional derivative of f at the point $(-3, 1, 1)$ in the direction of the point $(-2, 0, 0)$.
- What is the direction of maximum increase of the function f at the point $(-3, 1, 1)$?

2 (12 points) Evaluate or prove non-existence of the limit.

a.

$$\lim_{(x,y) \rightarrow (1,1)} \frac{x-y}{x^3-y^3}.$$

b.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^3 + y^3}.$$

c.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y}{x^3 + y^3}.$$

d.

$$\lim_{(x,y) \rightarrow (1,1)} \frac{x-y}{x^3 + y^3}.$$

3 (8 points) Let

$$f(x, y, z) = x^3 + yz^2$$

and

$$x = t^2 + s$$

$$y = t + s^2$$

$$z = st.$$

Use the chain rule to compute the partial derivative $\frac{\partial f}{\partial t}$ at $(s, t) = (-1, -1)$.

4 (8 points) Consider the surface S defined implicitly by the equation

$$ze^x + e^{z+1} = xy + y - 3.$$

- Find the equation of the tangent plane to S at the point $P = (0, 3, -1)$.
- Find the equation of the normal line to S at the point $P = (0, 3, -1)$.

5 (12 points) Find all local maxima, minima, and saddle points of

$$f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2.$$

6 (12 points) Find the absolute maximum and minimum of

$$f(x, y) = x^2 + 2y^2 - 2x + 3$$

in the region

$$x^2 + y^2 \leq 10.$$

7 (8 points) Evaluate the integral

$$\int_0^1 \int_y^1 ye^{x^3} dx dy.$$

8 (10 points) Find the volume of the solid bounded above by the surface

$$x^2 + y^2 + z^2 = 4$$

and below by the surface

$$z^2 = \frac{1}{3}(x^2 + y^2).$$

9 (10 points) Rewrite the integral

$$\int_0^6 \int_0^{6-x} \int_0^{6-x-y} dz dy dx$$

as an integral with respect to $dy dx dz$.

10 (10 points) Use integration to find the volume of the region

$$R := \left\{ (x, y, z) : x \geq 0, y \geq 0, z \geq 0, x + y \leq 4, z \leq \frac{1}{4}\sqrt{16 - y^2} \right\}.$$