

考試規範：

- ★ 本考試卷共八大題 (共二頁)，配分請見各題，合計 110 分。
- ★ 本考試禁止翻閱書籍、筆記，亦不得查詢任何資料。
- ★ 禁止使用電子設備 (如手機、計算機、電腦) 進行計算或查詢答案。
- ★ 所有解答須附詳細計算過程，否則不予計分。

1. Find the limit if it exists. Notice that L'Hôpital's Rule is forbidden.

(a) (5 pts) $\lim_{x \rightarrow 0} \frac{\tan x + 2x}{e^x + x}$

(b) (5 pts) $\lim_{h \rightarrow 0} \frac{2h}{\sqrt{5h+4}-2}$

(c) (5 pts) $\lim_{x \rightarrow 0} \frac{\sin(4x)}{\sin(5x) \cos(6x)}$

(d) (5 pts) $\lim_{t \rightarrow \infty} \frac{2-t}{2t + \cos t}$

(e) (5 pts) $\lim_{x \rightarrow 1} \frac{\cos\left(\pi x - \frac{\pi}{2}\right)}{x-1}$

2. (a) (5 pts) Given $y = \frac{1}{\cot x} + \left(\frac{1+3x}{3x}\right)(3-x) + \sqrt{x^7} + \ln 2$. Find the derivative $\frac{dy}{dx}$.

(b) (5 pts) Given $y = \pi^{\sqrt{x}} + x^{\sqrt{\pi}} + \left(\frac{\sin x}{1 + \cos x}\right)^2$. Find the derivative $\frac{dy}{dx}$.

(c) (5 pts) Given $y = \frac{(2-x)^{\frac{3}{2}}}{(1-x)^2 \sqrt{1-x^2}}$. Find the value of $\frac{dy}{dx}$ at $x = 0$.

(d) (5 pts) Given $f(x) = x^2 + 4x - 5$, $x < -2$. Find the value of $(f^{-1})'(0)$.

(e) (5 pts) Given $f(x) = \frac{(x-1)(x-2)(x-3)(x-4)}{(x-5)(x-6)(x-7)}$. Find the value of $f'(2)$.

3. (7 pts) Find an equation of the tangent line to the graph of

$$3 \tan^{-1}(2x) + \sin^{-1} y = \frac{11}{12} \pi$$

at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$.

4. Let $f(x) = \begin{cases} x^2 - x, & x \leq -1 \\ -2x^2 - 7x + a, & x > -1 \end{cases}$ be a continuous function on $\mathbb{R}$.

(a) (5 pts) Find the value of a .

(b) (8 pts) Use the definition of the derivative to determine whether (是否) f is differentiable at $x = -1$.

(請翻頁繼續作答 Please turn the page to continue answering)

5. Consider a function $f(x) = \frac{x^2 - 4}{x^4 - 16}$.

- (a) (2 pts) Find the domain of the function f .
- (b) (4 pts) Does the graph of f have vertical asymptote(s)? Given your reason!
- (c) (4 pts) Does the graph of f have horizontal asymptote(s)? Given your reason!

6. (10 pts) Find the absolute maximum and minimum of $f(x) = 10x(2 - \ln x)$ on $[1, e^2]$.

7. Suppose f is a function of x that satisfies the following two conditions

$$f(a+b) = f(a) + f(b) + a^2b + ab^2 \quad \forall a, b \in \mathbb{R} \quad \text{and} \quad \lim_{x \rightarrow 0} \frac{f(x)}{x} = 2025$$

- (a) (2 pts) Find the value of $f(0)$.
 - (b) (3 pts) Find the value of $f'(0)$.
 - (c) (5 pts) Find $f'(x)$.
8. (a) (5 pts) Show that $|\sin x| \leq |x|$ for all $x \in \mathbb{R}$.
(★ Hint: 分 $x = 0$ 和 $x \neq 0$ 的情況討論; 其中 $x = 0$ 的證明很簡單; $x \neq 0$ 的證明可使用均值定理)
- (b) (5 pts) Use the fact of (a) to find the limit $\lim_{x \rightarrow 0} \frac{\sin(x^2 \sin \frac{1}{x})}{x}$.

(考題結束 End of midterm exam questions)

Theorem (Mean Value Theorem 均值定理).

If f is continuous on $[a, b]$ and f is differentiable on (a, b) , then

$$\exists c \in (a, b) \text{ such that } f'(c) = \frac{f(b) - f(a)}{b - a}$$