

考試規範：

1. 本試卷共四大題(共一頁)，配分請見各題，合計 100 分。
2. 禁止帶手機、計算機及電腦等電子用品進入考場。
3. 所有題目的解答都要有計算過程，無計算過程不給分。

1. Let D be the plane region enclosed by the graph of curves $y = \sqrt{9 - x^2}$, $x = -1$, $y = 0$ and $y = x$. (設 D 是平面上由這些曲線的圖形圍繞而成的區域)

(1) (8 pts) Find the area of D .

(2) (6 pts) Find the volume of the solid generated by revolving D about x -axis. (求 D 繞 x 軸旋轉所生成之物體體積)

2. (6 pts) Find the area of the surface generated by revolving the curve $y = \sqrt{2x - x^2}$, $1 \leq x \leq 1.5$, about x -axis. (求曲線繞 x 軸旋轉所生成之曲面面積)

3. (8 pts×8) Evaluate the integrals. (計算積分)

(1) $\int_0^{\pi/2} \cos^3(3\theta) \sin^2(6\theta) d\theta$

(2) $\int x^3 \sin(2x) dx$

(3) $\int \frac{2x^2 - 6x + 4}{(x^2 + 1)(x - 1)^3} dx$

(4) $\int \tan^{-1}(\sqrt{x}) dx$

(5) $\int \sqrt[3]{\tan x} dx$

(6) $\int \ln(x) \cdot x^{\ln(x)-1} dx$

(7) $\int_0^{2025} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{2025 - x}} dx$

(8) $\int \frac{\sin^2(x)}{1 + \cos^2(x)} dx$

4. (8 pts×2) Find the limit if it exists. (求極限若存在)

(1) $\lim_{t \rightarrow (-2)^+} \int_t^2 \frac{5t + 10}{x^2 - x - 6} dx$

(2) $\lim_{x \rightarrow 0} \left(\csc^2(x) - \frac{1}{x^2} \right)$