

考試規範：

1. 本試卷共八大題(共一頁)，配分請見各題，合計 100 分。
2. 禁止帶手機、計算機及電腦等電子用品進入考場。
3. 所有題目的解答都要有計算過程，無計算過程不給分。

1. (6 points each) Evaluate or prove non-existence of the following limit.

(a) $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 - y^2}{\sqrt{x} - \sqrt{y}}$

(b) $\lim_{(x,y) \rightarrow (2,-1)} \frac{xy + 2}{x^2 - 4y^2}$

2. (10 points) Find the values of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of the equation $z^3 - xy + \sin(yz) + ye^x - 2 = 0$ at the point $(1, -1, 0)$.

3. Let $f(x, y, z) = \cos(xy) + e^{yz} + \ln(zx)$ and a point $P(1, 0, \frac{1}{2})$.

(a) (6 pts) Find the gradient (梯度) of f at the point P .

(b) (6 pts) Find the directional derivative (方向導數) at the point P in the direction of $\mathbf{v} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

(c) (4 pts) Find the directional derivative in the direction which the function **decrease** most rapidly (使函數下降最快的方向導數).

4. (12 points) Find the extreme values (極值) and saddle point (鞍點) of the function $f(x, y) = e^{-y}(x^2 + y^2)$.

5. (6 points each) Find (a) the tangent plane (切平面) and (b) the normal line (法線) of the surface $z = x \sec y - ye^x$ at the point $(0, 0, 0)$.

6. (10 points) Find the function $f(x, y, z)$ such that the gradient of f is

$$\nabla f = e^x \cos y \mathbf{i} - e^x \sin y \mathbf{j} + z \mathbf{k}$$

and satisfies $f(0, \frac{\pi}{2}, 1) = \frac{1}{2}$.

7. (6 points each) Evaluate the following integrals.

(a) $\int_0^{\ln 4} \int_0^{\sqrt{2}} \int_{-1}^1 ye^z dx dy dz$

(b) $\int_0^1 \int_{\sqrt[3]{y}}^1 \frac{3\pi \sin(\pi x^3)}{x} dx dy$

(c) $\iint_R \frac{\ln(x^2 + y^2)}{\sqrt{x^2 + y^2}} dA$, where $R : 1 \leq x^2 + y^2 \leq e$.

8. (10 points) Use cylindrical coordinate (圓柱坐標系) to find the volume of the solid bounded above by $z = 2x$ and below by $z = 2x^2 + 2y^2$.