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Fast SDRE Based on SDA for Exoskeleton

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Abstract

With the rapid development of military and medical, exoskeleton which helps the wearer's movements plays an important role. To achieve the specified action precisely and effectively is a significant issue. Here, we adopt the optimal regulator state-dependent Riccati equation (SDRE) to control the exoskeleton. Besides, a state-of-the-art CARE solver the Structure-Preserving double Algorithm (SDA) is introduced and 4-5 times faster than "icare" in MATLAB. It significantly improves SDRE calculation efficiency and obtains more accurate values.

Problem description

Given a target trajectory, the exoskeleton can make the desired action by control strategy SDRE. However, it needs to solve CARE at every step, resulting in a huge computational time using traditional CARE solvers and making it impossible to achieve the expected action immediately. The bottleneck is handled by the Structure-Preserving double Algorithm (SDA).

Dynamics System

The bionic leg platform is modeled as a double pendulum system.
Based on Euler-Lagrange equation, the dynamic equation is as follows:

$$\tau = M(\theta)\ddot{\theta} + V(\theta,\dot{\theta})\dot{\theta} + G(\theta)$$

where $\theta = [\theta_1\ \theta_2]^T$ state vector and τ is the control input

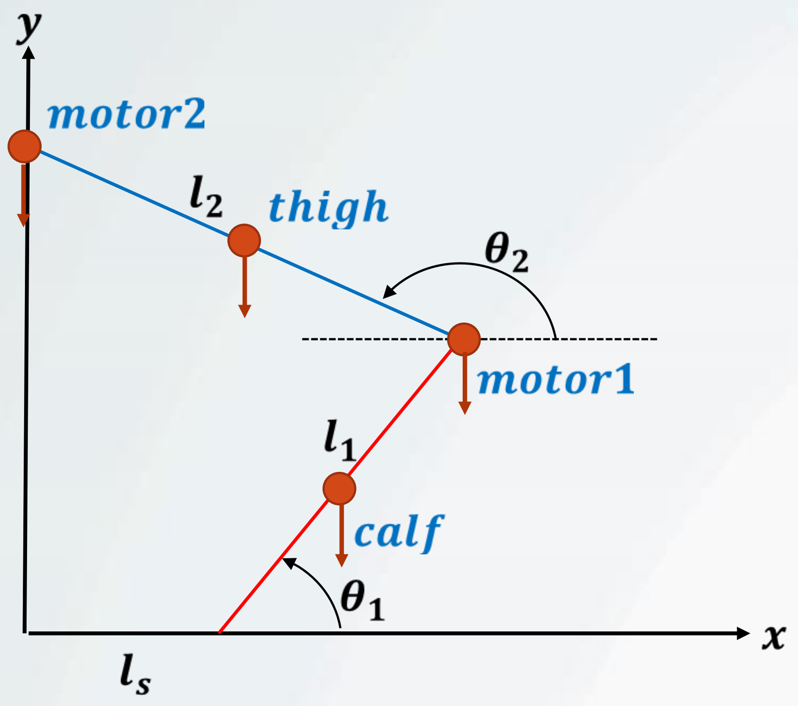

Figure 1. Bionic leg platform model

Target angle function

To simulate squatting and standing up, the waist position will be moving up and down the Y axis, so the constraint condition is obtained.

Constraint： $l_s + l_1\cos(\theta_1) + l_2\cos(\theta_2) = 0$

$\theta_{1f}(t) = \alpha\cos(2t) + C$ (deg) $\quad (\alpha = 0.2\ C = \theta_1(0) - \alpha)$

$\dot{\theta}_{1f}(t) = -2\alpha\sin(2t)$

$\theta_{2f}(t)$ and $\dot{\theta}_{2f}(t)$ can be obtained by the constraint condition.

SDRE scheme

SDRE scheme contains the type of nonlinear system in the forms and minimizes the cost function :

$$\dot{x} = f(x) + B(x)u \ \left(\dot{x} = A(x)x + B(x)u\right)$$

$$\min J = \frac{1}{2}\int_0^\infty [x^T Q x + u^T R u]dt$$

where $x = \left[\theta - \theta_f\ \ \dot{\theta} - \dot{\theta}_f\right]^T$ error state vector.

The optimal solution for control is

$$u = -Kx \qquad \text{with } K = -R^{-1}B(x)X(x) \qquad G(x) = B(x)R^{-1}B^T(x)$$

where $X(x)$ is solution for Continuous Algebraic Riccati Equation

$$A^T(x)X(x) + X(x)A(x) - X(x)G(x)X(x) + Q = 0$$

Computation of the solution X to CARE

SDA (Algorithm 1)

Input the Hamiltonian matrix $\mathcal{H} = \begin{bmatrix} A & G \\ Q & -A^T \end{bmatrix}$ with $\sigma(\mathcal{H}) \cap \mathrm{Im} = \phi$;)

Output the symmetric positive semi-definite solution X to CARE

Find a suitable value $\gamma>0$ and let $W_\gamma = A_\gamma^T + QA_\gamma^{-1}G$ and $A_\gamma = A - \gamma I$

Initialize k=0, $A_0 = I + 2\gamma W_\gamma^{-T}, G_0 = 2\gamma A_\gamma^{-1}GW_\gamma^{-1}, Q_0 = 2\gamma W_\gamma^{-1}\,QA_\gamma^{-1}$

For k=0,1, 2,……

$$A_{k+1} = A_k(I + G_kQ_k)^{-1}A_k$$

$$G_{k+1} = G_k + A_k(I + G_kQ_k)^{-1}G_kA_k^T$$

$$Q_{k+1} = H_k + A_k^TQ_k(I + G_kQ_k)^{-1}A_k$$

Break when $||Q_k - Q_{k-1}|| \leqq \varepsilon||Q_k||$

Set $X \leftarrow Q_k$ is the solution.

End

Result and conclusion

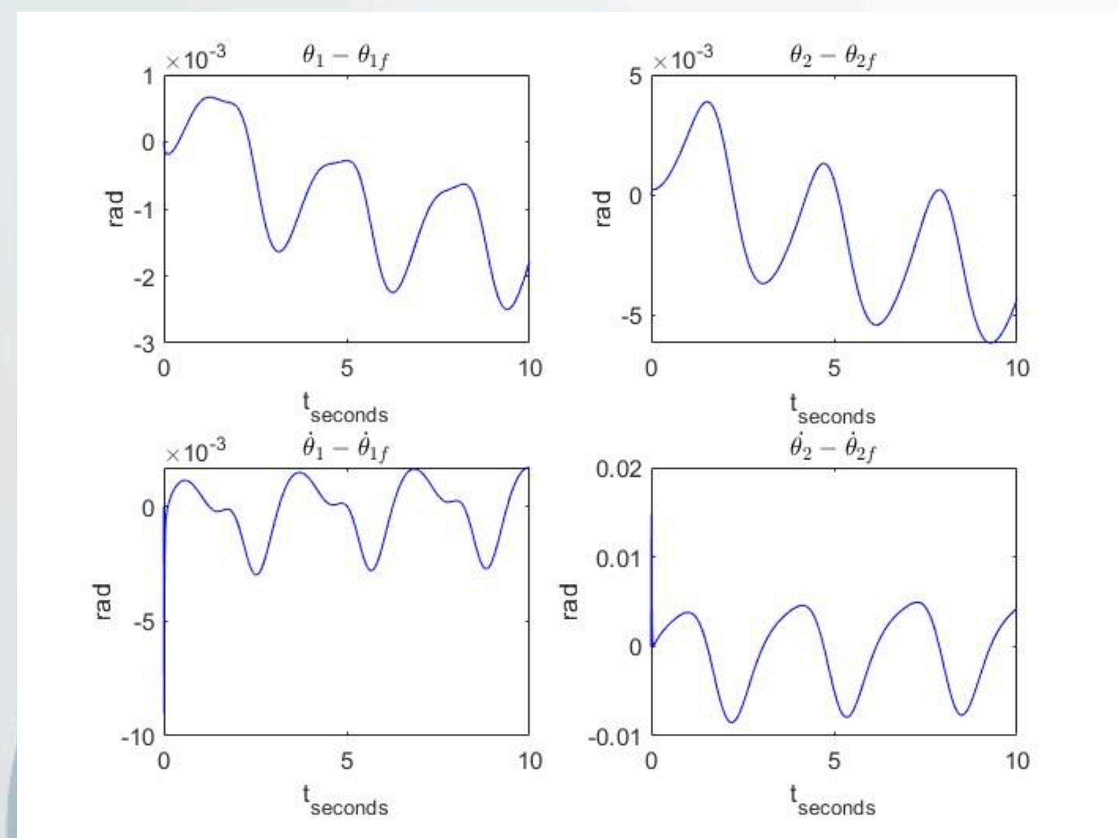

Figure 2. SDRE control for error state.

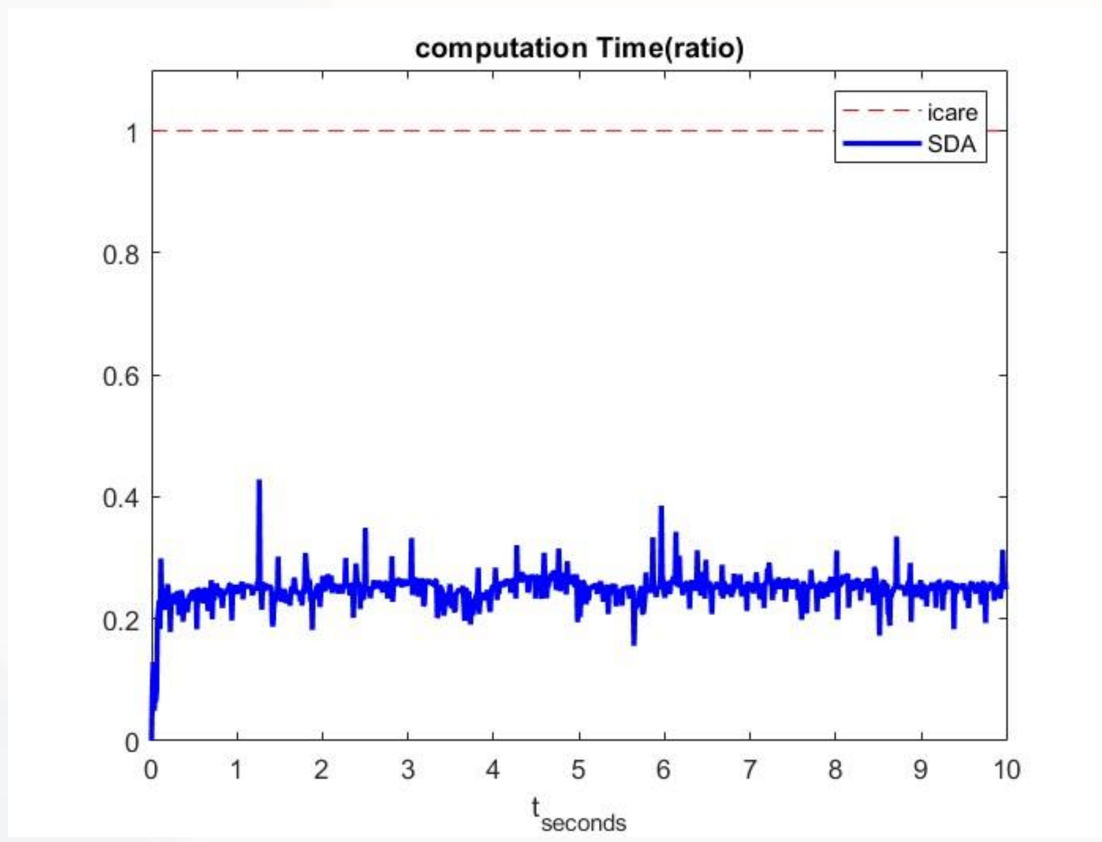

Figure 3. Computation Time between SDA and icare

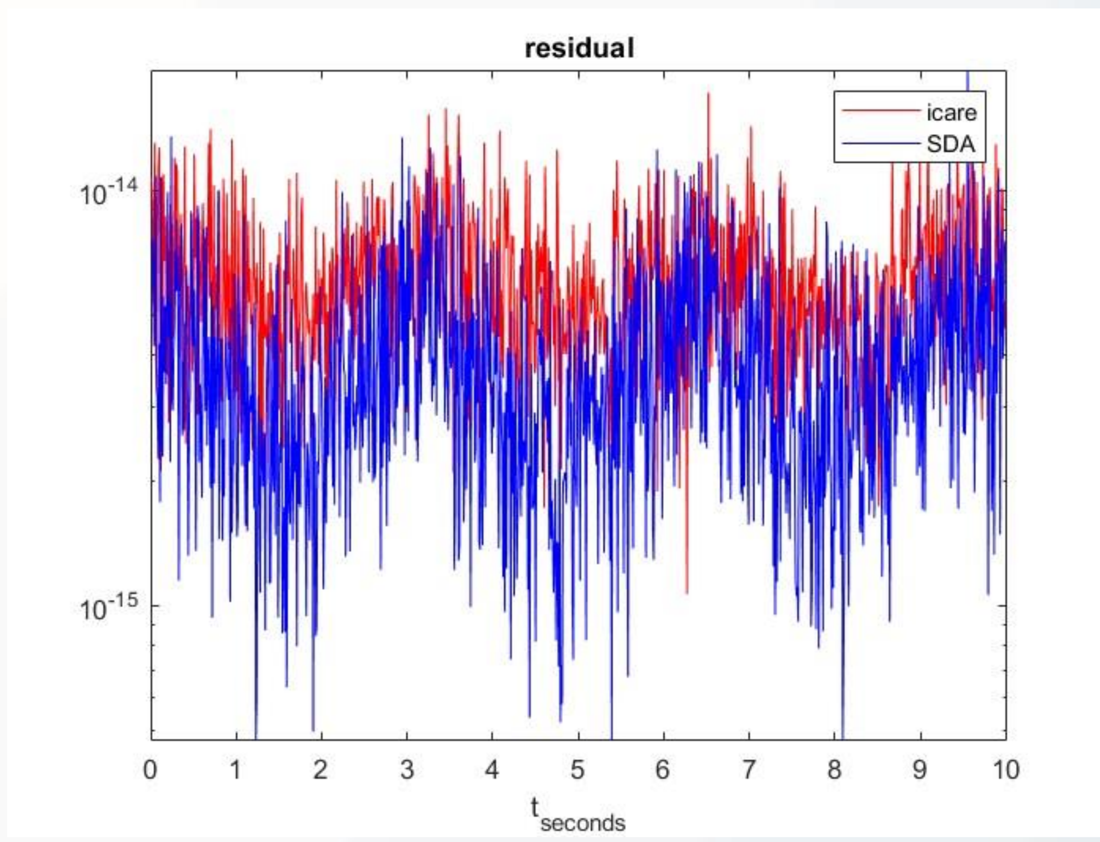

Figure 4. Residual between SDA and icare

Observation that the error is small and tends to zero, it means that SDRE has great control performance. Besides, we compare SDA and "icare" for computation time and residual. The average time of SDA and "icare" are respectively $7.642 * 10^{-5}$s and $3.54 * 10^{-4}$s and the residual of SDA and "icare" are respectively $3.961 * 10^{-15}$ and $6.203 * 10^{-15}$. It shows that SDA is 4-5 times faster than icare and has more accurate solution. Hence, SDA greatly reduces the computation of SDRE.

Reference

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