2023

【海報論文】

☐一般組 ☐研究生組

☐大學組

11th 台灣 TAIWAN 工業與應用數學會年會

TWSIAM Annual Meeting

Investigating the performance of Level Set Method for Image Segmentation

Tsu-Yin Chou (周姿吟), *Department of Mathematics, National Central University, Taiwan* (ziyin91123@gmail. com)

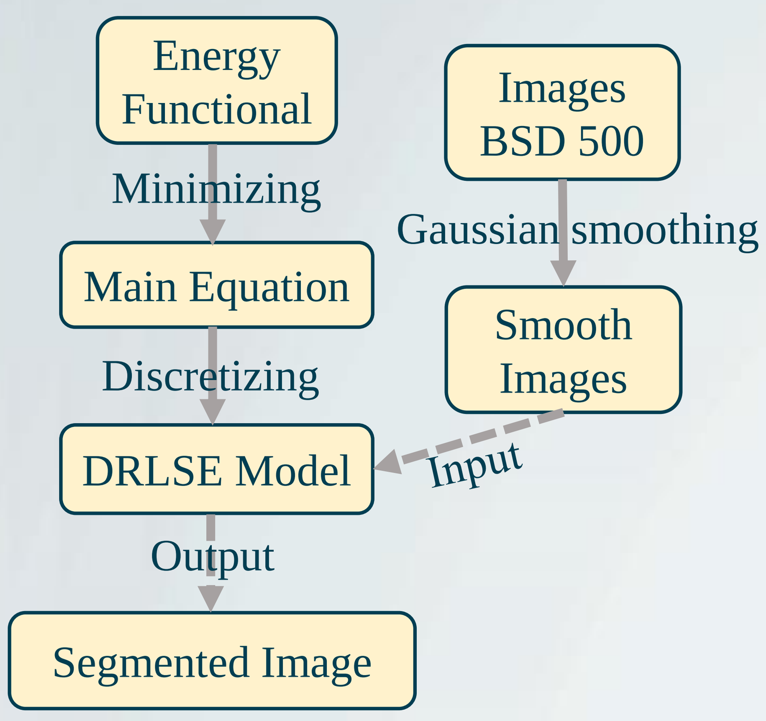
Advisor: Chung-Yu Shih (施重宇) , Prof. Shen-Fu Tsai (蔡昇甫)

Abstract

Level set methods are widely used in image processing and computer vision for image segmentation, which involves dividing a digital image into multiple segments. The Distance Regularized Level Set Evolution (DRLSE) model [1] introduces distance regularization to avoid reinitialization, which was previously proposed to address numerical instabilities, as reinitialization requires a lot of computation. The DRLSE model partitions an image into two parts based on the zero level contour. By modifying the ratio between the length and area of the level set function (LSF), we discuss the geometric meaning of the DRLSE model and show that it performs well on circular objects. We also propose a suitable initial LSF to address the issue of sensitivity and set a stopping condition to maintain the desired shape of the LSF during evolution. The results demonstrate that our remedy can address the rounding error issue to some degree.

Problem description

Many research studies [2] [3] use variational methods for image segmentation with an available energy functional. The functional is minimized and transformed into an ODE. The procedure is illustrated in the flowchart below.



- **Minimizing the energy functional**

We aim to minimize an energy functional [1] that incorporates a level set regularization term, a contour length term, and a term for the area enclosed by the contour. Let φ be the LSF.

$$\mathcal{E}(\varphi)= \mu \int_\Omega p(|\nabla\varphi|)dx+\lambda\int_\Omega g\delta(\varphi)|\nabla\varphi|dx+\alpha\int_\Omega gH(-\varphi)dx$$

where $p(s) = \begin{cases} \frac{1}{(2\pi)^2}(1-\cos(2\pi s)), & s \le 1 \\ \frac{1}{2}(s-1)^2, & s \ge 1 \end{cases}$ and $g \triangleq \frac{1}{1+|\nabla G_\sigma * I|^2}$

Note that G_σ is a Gaussian kernel with a deviation σ and I is the image.

$$H(x) = \begin{cases} \frac{1}{2}(1+\frac{x}{2}+\frac{1}{\pi}\sin(\frac{\pi x}{2})), & |x| \le 2 \\ 1, & x > 2 \\ 0, & x < -2 \end{cases} \quad \text{and} \quad \delta(x)= H'(x)$$

To obtain the main ODE as below, the energy functional above is minimized and we use the gradient flow equation [4]:

$$\frac{\partial\varphi}{\partial t} = \mu\, div(d_p(|\nabla\varphi|)\nabla\varphi)+\lambda\, \delta(\varphi)div(g\frac{\nabla\varphi}{|\nabla\varphi|}) +\alpha\, g\delta(\varphi)$$

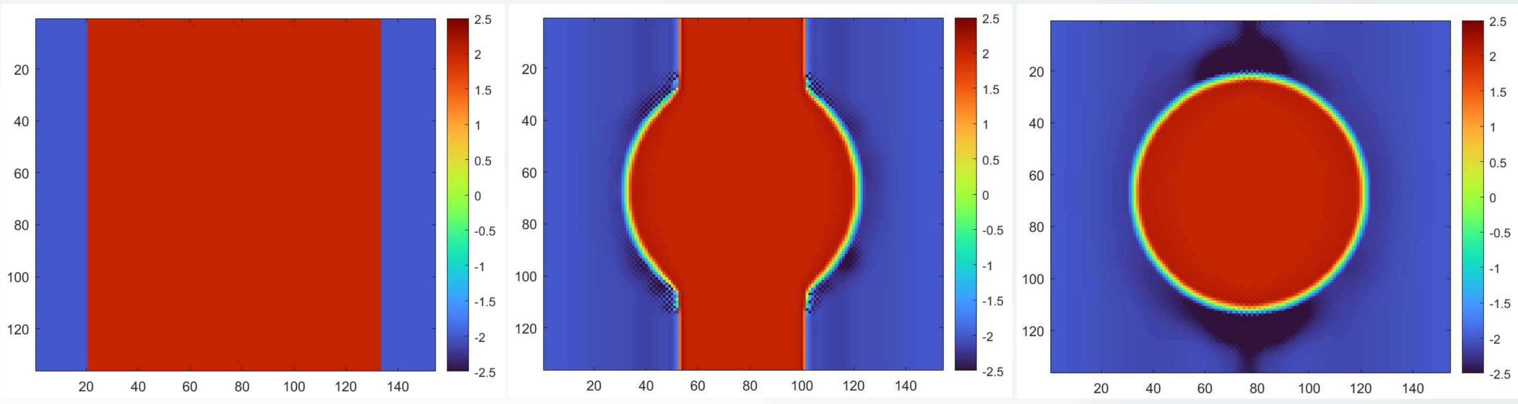
where $d_p(s) = p'(s)/s$

- **Discretizing the main equation**

The main equation in time is discretized using forward-Euler method, which leads to the key step in the DRLSE model:

$$\frac{\varphi_n-\varphi_{n-1}}{\Delta t} = \mu\, div(d_p(|\nabla\varphi_{n-1}|)\nabla\varphi_{n-1})+\lambda\, \delta(\varphi_{n-1})div(g\frac{\nabla\varphi_{n-1}}{|\nabla\varphi_{n-1}|}) +\alpha\, g\delta(\varphi_{n-1})$$

The first term is the distance regularization term, satisfying the CFL condition, $\mu\Delta t < 0.25$. The second and third terms are the length and area terms, respectively. The initial LSF is crucial in the DRLSE model, which influences the segmentation performance. The evolution of the LSF is illustrated below.



- **Gaussian smoothing images**

We preprocess images by converting them to gray level followed by Gaussian smoothing. Note that BSD500 is a dataset.

Results and discussion

- **Necessary for Gaussian smoothing**

The advantage of a Gaussian filter is that it can smooth images, which prevents the strong effects of the first term in our DRLSE model. In 1(d), without Gaussian filter, the LSF could not maintain a desired shape.

1(a) 1(b) 1(c) 1(d)

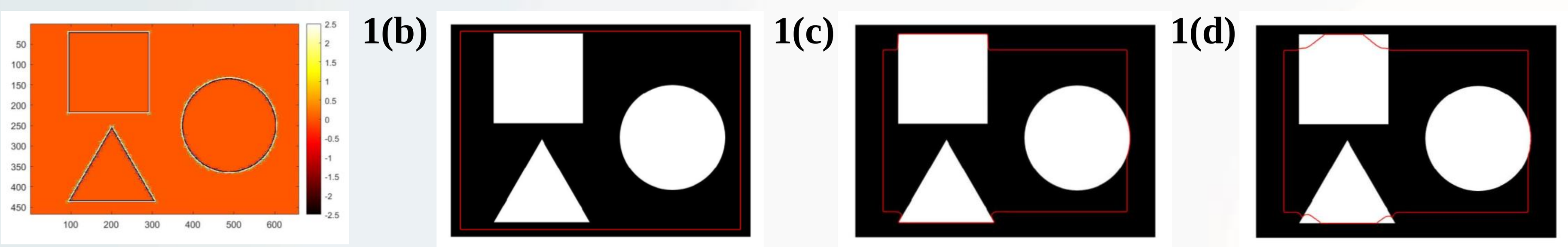


Fig 1. Resolution: 467 × 659. Set $\Delta t = 2$, $\mu = 0.24/\Delta t$, $\lambda = 3$, $\alpha = 3$, binary step $c_0 = 2$. (a) The difference between the original image and the image after Gaussian smoothing. (b) initial LSF. (c) 100 iterations with Gaussian filter. (d) 100 iterations without Gaussian filter.

2(a) 2(b) 2(c) 2(d)

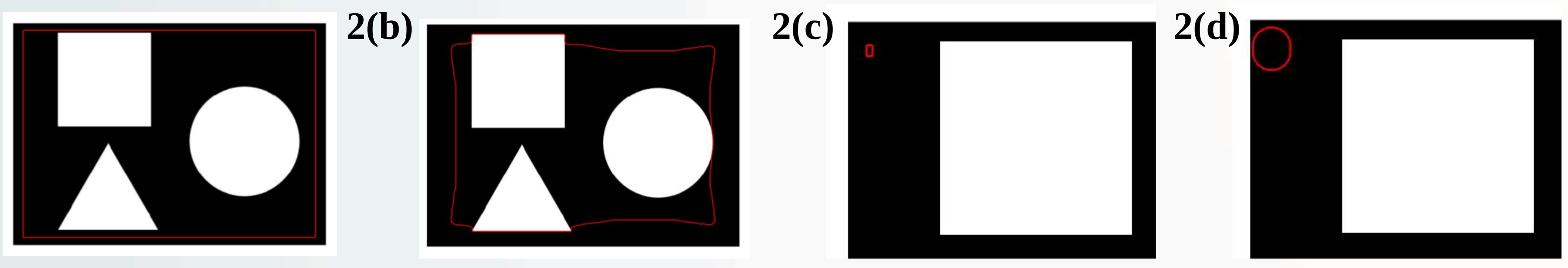


Fig 2. Resolution: 467 × 659. Set $\Delta t = 2$, $\mu = 0.24/\Delta t$, binary step $c_0 = 2$. (a) initial LSF. (b) Set $\lambda = 5$, $\alpha = 3$. 100 iterations. (c) initial LSF. (d) Set $\lambda = 3/4$, $\alpha = -3$. 40 iterations.

3(a) 3(b) 3(c) 3(d)

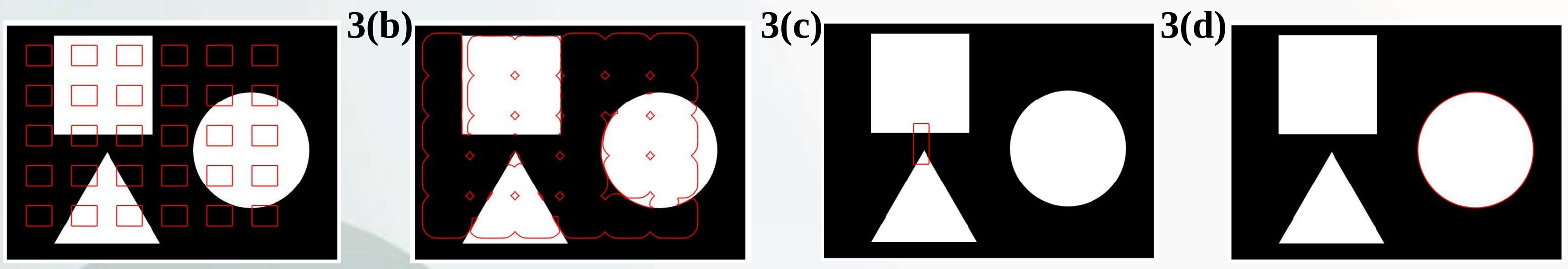


Fig 3. Resolution: 467 × 659. Set $\Delta t = 2$, $\mu = 0.24/\Delta t$, binary step $c_0 = 2$. (a) initial LSF. (b) Set $\lambda = 5$, $\alpha = -3$. 60 iterations. (c) initial LSF. (d) Set $\lambda = 5$, $\alpha = -3$. 1500 iterations.

- **Geometric meaning**

In 2(b), the length term dominates with a twisted contour as its geometric meaning, while in 2(d), the area term dominates with a circular contour as its geometric meaning. The length term and area term have distinct mathematical meanings.

- **Limitation of initial LSF**

In 3(b), the evolution caused rectangles to combine and expand out of the image. In 3(d), a circle appeared in the segmentation result that was not present in the initial LSF. Therefore, the DRLSE model is sensitive to the initial LSF. Fortunately, we found a proper initial LSF that can handle many cases.

- **Remedy for rounding error**

A large number of iterations can lead to unstable segmentation results, as shown in Fig. 4(b). To avoid this, we propose a stopping condition based on the 2-norm of the difference between previous and new evolutions. If the 2-norm is less than a given tolerance, we stop the iterations. Note that the tolerance needs to be chosen carefully.

4(a) 4(b) 4(c)

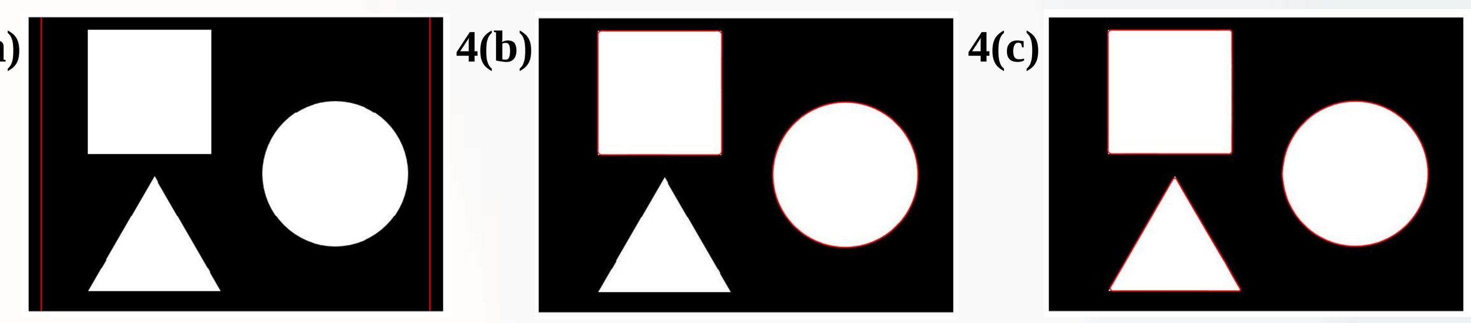


Fig 4. Resolution: 467 × 659. Set $\Delta t = 2$, $\mu = 0.24/\Delta t$, binary step $c_0 = 2$, $\lambda = 1$, $\alpha = -2$. (a) proper initial LSF. (b) 2300 iterations. (c) Set tolerance = 10, after 990 iterations.

- **Exhibition**

5(a) 5(b) 5(c)

Fig 5. Resolution: 319 × 383. Set $\Delta t = 2$, $\mu = 0.24/\Delta t$, binary step $c_0 = 2$, $\lambda = 3$, $\alpha = -3$. (a) original image. (b) proper initial LSF. (c) Set tolerance = 5, after 700 iterations.

6(a) 6(b) 6(c)

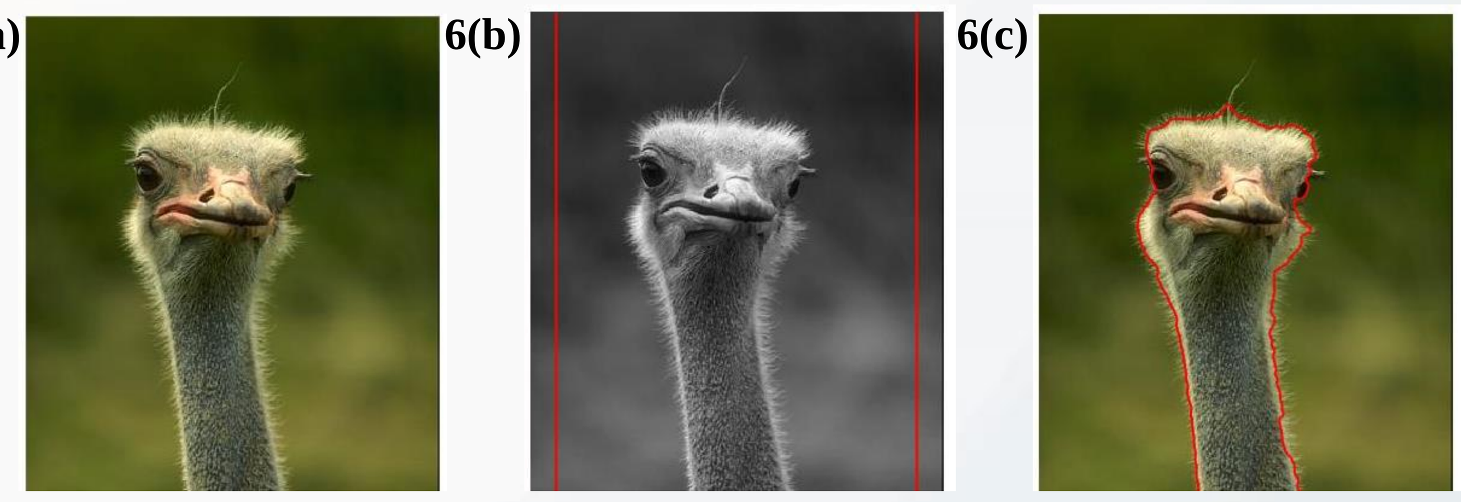


Fig 6. Resolution: 481 × 321. Set $\Delta t = 2$, $\mu = 0.24/\Delta t$, binary step $c_0 = 2$, $\lambda = 5$, $\alpha = -3$. (a) original image. (b) proper initial LSF. (c) Set tolerance = 10, after 490 iterations.

References

[1] Li, Chunming, et al. Distance regularized level set evolution and its application to image segmentation. *IEEE transactions on image processing*, Vol. 19, No. 12, p. 3243-3254 (2010)

[2] G. Aubert and P. Kornprobst, *Mathematical Problems in Image Processing: Partial Differential Equations and the Calculus of Variations.* New York: Springer-Verlag (2002)

[3] Chan, Tony F., and Luminita A. Vese. Active contours without edges. *IEEE Transactions on image processing*, Vol. 10, No. 2, p. 266-277 (2001)

[4] Li, Chunming, et al. Level set evolution without re-initialization: a new variational formulation. *IEEE computer society conference on computer vision and pattern recognition*, Vol. 1, p. 430-436 (2005)

Conclusions

- The Gaussian filter can smooth images, preventing the distance regularization term from significantly affecting the LSF evolution and maintaining the desired LSF shape.
- The length and area terms have geometric meanings, and setting a small length and relatively large area can successfully segment circular objects.
- The DRLSE model is sensitive to its initial LSF, but a rectangle slightly smaller than the image could be a suitable initial LSF.
- Our method can partially address the rounding error issue, but in practice there could still be circumstances when the level set method does not produce desired segmentation.