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Analysis of seepage problems with sheet plies by using the meshfree boundary integral equation method

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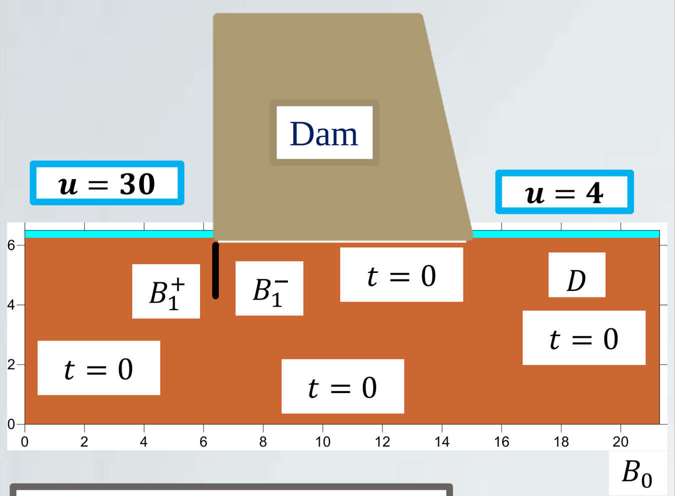
Abstract

Regarding the problem of seepage under a dam with a sheet pile, the meshfree boundary integral equation method (BIEM) in conjunction with the dual formulation is employed to solve the field of velocity potential. Different from the conventional boundary element method (BEM), the present method is free of mesh generation. After using the parametric form to represent the boundary contour and adopting the Gaussian quadrature, only collocating points on the boundary is required to obtain the linear algebraic equations. By introducing the local exact solution, the singular integral in the sense of the Cauchy principal value can be novelty determined. Since the local exact solution for the ordinary boundary is unsuitable for the degenerate boundary. Therefore, the local exact solution on the degeneration boundary is rederived by using the elliptical coordinates. Finally, the three cases are considered to examine the effect of the sheet pile.

Problem description



Governing equation：

Laplace equation： $\nabla^2 u(\mathbf{x}) = 0 , \mathbf{x} \in D$

u is the velocity potential

$$t = \frac{\partial u}{\partial n}$$

Meshfree boundary integral equation method

$$0 = \int_B T(\mathbf{s},\mathbf{x})[u(\mathbf{s}) - w(\mathbf{s})]dB(\mathbf{s}) - \int_B U(\mathbf{s},\mathbf{x})[t(\mathbf{s}) - w_p(\mathbf{s})]dB(\mathbf{s}),\ \mathbf{x} \in B$$

$$0 = \int_B M(\mathbf{s},\mathbf{x})[u(\mathbf{s}) - w(\mathbf{s})]dB(\mathbf{s}) - \int_B L(\mathbf{s},\mathbf{x})[t(\mathbf{s}) - w_p(\mathbf{s})]dB(\mathbf{s}),\ \mathbf{x} \in B$$

$$t(\mathbf{s}) = \frac{\partial u(\mathbf{s})}{\partial n(\mathbf{s})} \qquad w_p(\mathbf{s}) = \frac{\partial w(\mathbf{s})}{\partial n(\mathbf{s})} \qquad B = B_0 \cup B_1 \qquad B_1 = B_1^+ \cup B_1^-$$

Ordinary boundary

For $\mathbf{x} \in B_o$

$$0 = \int_B T(\mathbf{s},\mathbf{x})[u(\mathbf{s}) - w(\mathbf{s})]dB(\mathbf{s}) - \int_B U(\mathbf{s},\mathbf{x})[t(\mathbf{s}) - w_p(\mathbf{s})]dB(\mathbf{s}),\ \mathbf{x} \in B_o$$

$$w(\mathbf{s}) = u(\mathbf{x}) + [(s_x - x)n_x(\mathbf{x}) + (s_y - y)n_y(\mathbf{x})]t(\mathbf{x}).$$

$$0 = \sum_{j=1}^{N_g} \omega_j T(\mathbf{s}_j,\mathbf{x}_i)[u_j - w_j^i] j_b(\xi_j) - \sum_{j=1}^{N_g} \omega_j U(\mathbf{s}_j,\mathbf{x}_i)[t_j - w_{p\ j}^i] j_b(\xi_j)$$

$$j_b(\xi) = \left(\frac{\tau_L - \tau_0}{2}\right)\sqrt{(x'(\tau))^2 + (y'(\tau))^2}\Big|_{\tau=\tau(\xi)}$$

Degenerate boundary

For $\mathbf{x} \in B_1$

$\mathbf{x}^+$ B_1^+ / B_1^-

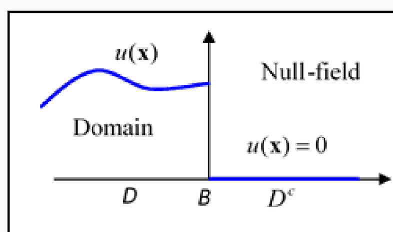
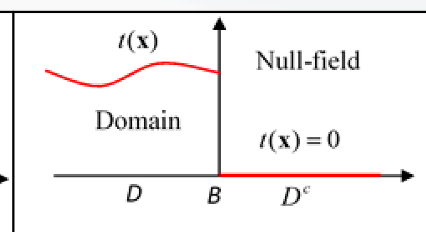
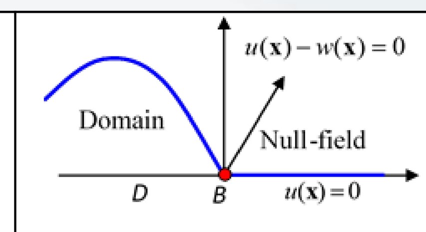
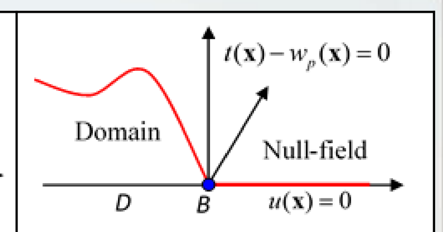
$$0 = \int_{B_1} T(\mathbf{s},\mathbf{x})\,[u(\mathbf{s}) - w(\mathbf{s})]dB(\mathbf{s}) - \int_{B_1} U(\mathbf{s},\mathbf{x})\,[t(\mathbf{s}) - w_p(\mathbf{s})]dB(\mathbf{s}) + \int_{B_0} T(\mathbf{s},\mathbf{x})\,u(\mathbf{s})dB(\mathbf{s}) - \int_{B_0} U(\mathbf{s},\mathbf{x})\,t(\mathbf{s})dB(\mathbf{s}),\ \mathbf{x} \in B_1^+$$

$$w(\mathbf{s}) = f_1(\mathbf{s},\mathbf{x}^+)u(\mathbf{x}^+) + f_2(\mathbf{s},\mathbf{x}^+)u(\mathbf{x}^-) + g_1(\mathbf{s},\mathbf{x}^+)t(\mathbf{x}^+) + g_2(\mathbf{s},\mathbf{x}^+)t(\mathbf{x}^-)$$

$$0 = \int_{B_1} M(\mathbf{s},\mathbf{x})\,[u(\mathbf{s}) - w(\mathbf{s})]dB(\mathbf{s}) - \int_{B_1} L(\mathbf{s},\mathbf{x})\,[t(\mathbf{s}) - w_p(\mathbf{s})]dB(\mathbf{s}) + \int_{B_0} M(\mathbf{s},\mathbf{x})\,u(\mathbf{s})dB(\mathbf{s}) - \int_{B_0} L(\mathbf{s},\mathbf{x})\,t(\mathbf{s})dB(\mathbf{s}),\ \mathbf{x} \in B_1^+$$

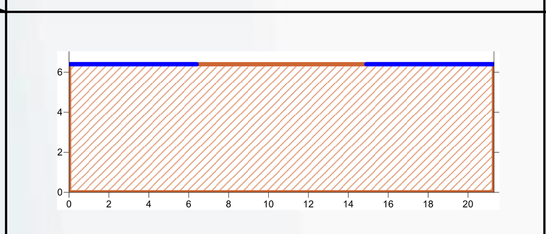
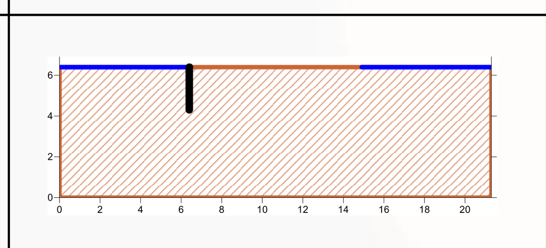
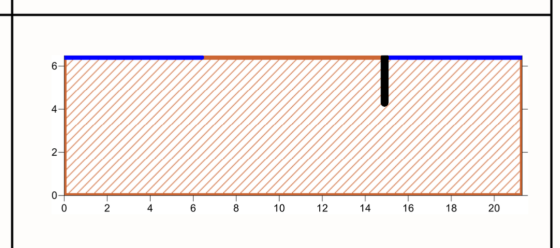
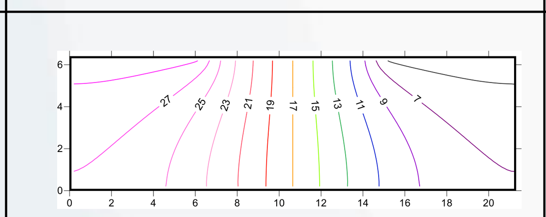
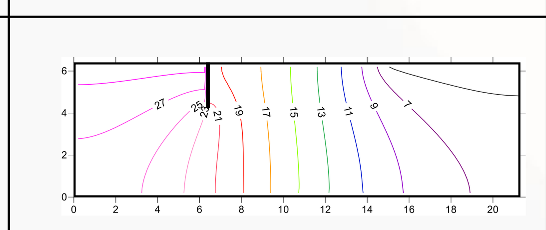
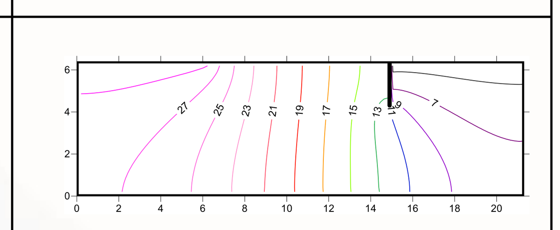
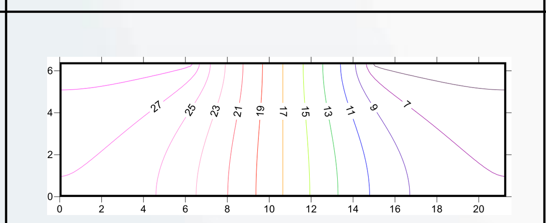
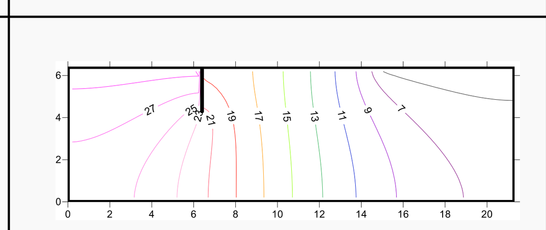
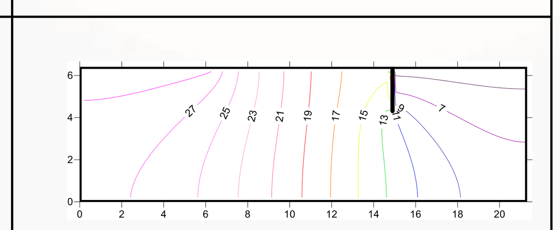
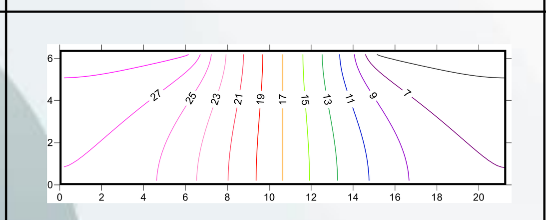
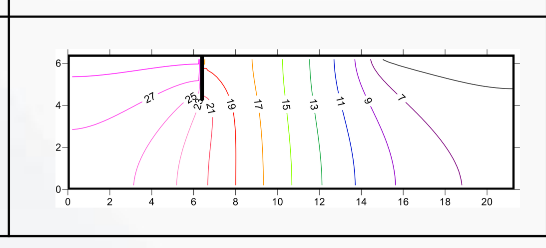
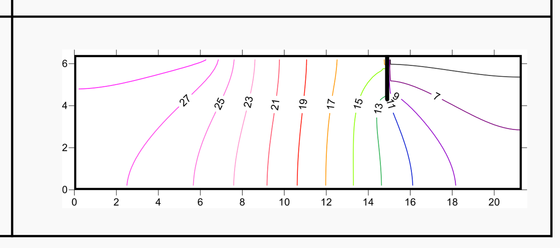
$$w(\mathbf{s}) = f_1(\mathbf{s},\mathbf{x}^-)u(\mathbf{x}^+) + f_2(\mathbf{s},\mathbf{x}^-)u(\mathbf{x}^-) + g_1(\mathbf{s},\mathbf{x}^-)t(\mathbf{x}^+) + g_2(\mathbf{s},\mathbf{x}^-)t(\mathbf{x}^-)$$

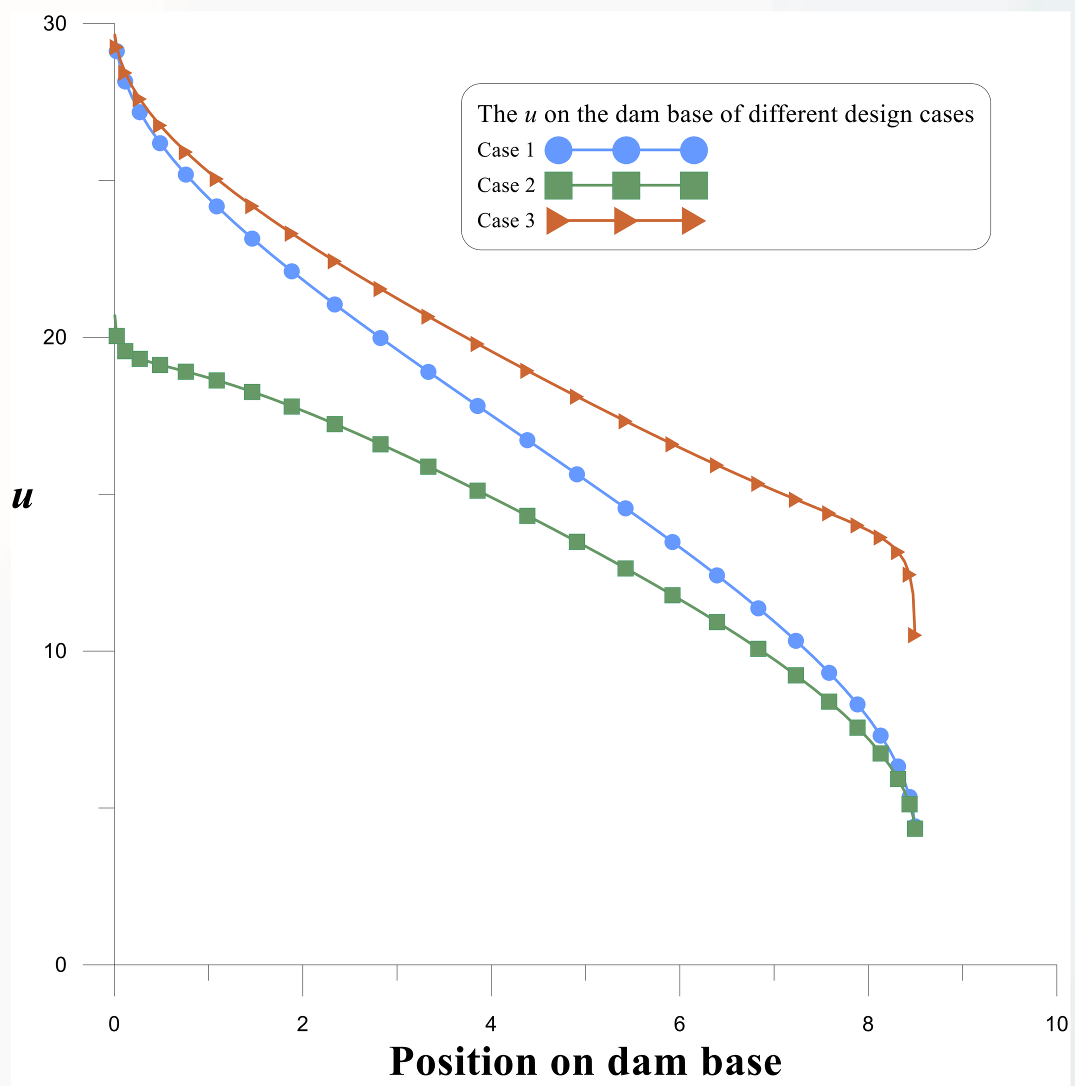
$\mathbf{x}^-$ B_1^+ / B_1^-

			
Potential field $u(\mathbf{x})$	Normal derivative of the potential field $t(\mathbf{x})$	$u(\mathbf{x}) - w(\mathbf{x})$	$t(\mathbf{x}) - w_p(\mathbf{x})$
Discontinuity behavior of the field across the boundary from the domain to the complementary domain		Continuous behavior of the function across the boundary from the domain to the complementary domain	

Results and discussion

$f_1(\mathbf{s},\mathbf{x}^+) = \frac{e^{\xi_x}}{\cos(\eta_{x^+})}e^{-\xi_s}\cos(\eta_s) - \frac{1}{2}\frac{e^{2\xi_x}}{\cos(2\eta_{x^+})}e^{-2\xi_s}\cos(2\eta_s) + \frac{e^{\xi_x}}{\sin(\eta_{x^+})}e^{-\xi_s}\sin(\eta_s) - \frac{1}{2}\frac{e^{2\xi_x}}{\sin(2\eta_{x^+})}e^{-2\xi_s}\sin(2\eta_s)$	$f_2(\mathbf{s},\mathbf{x}^+) = \frac{e^{\xi_x}}{\cos(\eta_{x^+})}e^{-\xi_s}\cos(\eta_s) - \frac{1}{2}\frac{e^{2\xi_x}}{\cos(2\eta_{x^+})}e^{-2\xi_s}\cos(2\eta_s) - \frac{e^{\xi_x}}{\sin(\eta_{x^+})}e^{-\xi_s}\sin(\eta_s) + \frac{1}{2}\frac{e^{2\xi_x}}{\sin(2\eta_{x^+})}e^{-2\xi_s}\sin(2\eta_s)$	$g_1(\mathbf{s},\mathbf{x}^+) = -\frac{1}{2}J(\mathbf{x}^+)[\frac{e^{\xi_x}}{\cos(\eta_{x^+})}e^{-\xi_s}\cos(\eta_s) - \frac{e^{2\xi_x}}{\cos(2\eta_{x^+})}e^{-2\xi_s}\cos(2\eta_s) + \frac{e^{\xi_x}}{\sin(\eta_{x^+})}e^{-\xi_s}\sin(\eta_s) - \frac{e^{2\xi_x}}{\sin(2\eta_{x^+})}e^{-2\xi_s}\sin(2\eta_s)]$	$g_2(\mathbf{s},\mathbf{x}^+) = -\frac{1}{2}J(\mathbf{x}^+)[\frac{e^{\xi_x}}{\cos(\eta_{x^+})}e^{-\xi_s}\cos(\eta_s) - \frac{e^{2\xi_x}}{\cos(2\eta_{x^+})}e^{-2\xi_s}\cos(2\eta_s) - \frac{e^{\xi_x}}{\sin(\eta_{x^+})}e^{-\xi_s}\sin(\eta_s) + \frac{e^{2\xi_x}}{\sin(2\eta_{x^+})}e^{-2\xi_s}\sin(2\eta_s)$
$f_1(\mathbf{s},\mathbf{x}^-) = \frac{e^{\xi_x}}{\cos(\eta_{x^-})}e^{-\xi_s}\cos(\eta_s) - \frac{1}{2}\frac{e^{2\xi_x}}{\cos(2\eta_{x^-})}e^{-2\xi_s}\cos(2\eta_s) + \frac{e^{\xi_x}}{\sin(-\eta_{x^-})}e^{-\xi_s}\sin(\eta_s) - \frac{1}{2}\frac{e^{2\xi_x}}{\sin(-2\eta_{x^-})}e^{-2\xi_s}\sin(2\eta_s)$	$f_2(\mathbf{s},\mathbf{x}^-) = \frac{e^{\xi_x}}{\cos(\eta_{x^-})}e^{-\xi_s}\cos(\eta_s) - \frac{1}{2}\frac{e^{2\xi_x}}{\cos(2\eta_{x^-})}e^{-2\xi_s}\cos(2\eta_s) - \frac{e^{\xi_x}}{\sin(-\eta_{x^-})}e^{-\xi_s}\sin(\eta_s) + \frac{1}{2}\frac{e^{2\xi_x}}{\sin(-2\eta_{x^-})}e^{-2\xi_s}\sin(2\eta_s)$	$g_1(\mathbf{s},\mathbf{x}^-) = -\frac{1}{2}J(\mathbf{x}^-)[\frac{e^{\xi_x}}{\cos(\eta_{x^-})}e^{-\xi_s}\cos(\eta_s) - \frac{e^{2\xi_x}}{\cos(2\eta_{x^-})}e^{-2\xi_s}\cos(2\eta_s) + \frac{e^{\xi_x}}{\sin(-\eta_{x^-})}e^{-\xi_s}\sin(\eta_s) - \frac{e^{2\xi_x}}{\sin(-2\eta_{x^-})}e^{-2\xi_s}\sin(2\eta_s)]$	$g_2(\mathbf{s},\mathbf{x}^-) = -\frac{1}{2}J(\mathbf{x}^-)[\frac{e^{\xi_x}}{\cos(\eta_{x^-})}e^{-\xi_s}\cos(\eta_s) - \frac{e^{2\xi_x}}{\cos(2\eta_{x^-})}e^{-2\xi_s}\cos(2\eta_s) - \frac{e^{\xi_x}}{\sin(-\eta_{x^-})}e^{-\xi_s}\sin(\eta_s) + \frac{e^{2\xi_x}}{\sin(-2\eta_{x^-})}e^{-2\xi_s}\sin(2\eta_s)]$

	Case 1	Case 2	Case 3
Sketch of the seepage problem			
Meshfree BIEM			
Dual BEM			
FEM			



Conclusions

1. We successfully applied the meshfree BIEM to the problem of seepage under a dam with a sheet pile.
2. To solve the degeneration problem arising from a degenerate boundary, it is necessary to use the hypersingular boundary integral equation.
3. The appropriate local exact solution for the problem with a degenerate boundary can be derived by using the elliptical coordinates.
4. Form the viewpoint of stability of dam, the case2 is the better choice than other two cases.

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