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Conduction shape factor in exchanger tubes with a slit by using the meshfree boundary integral equation method

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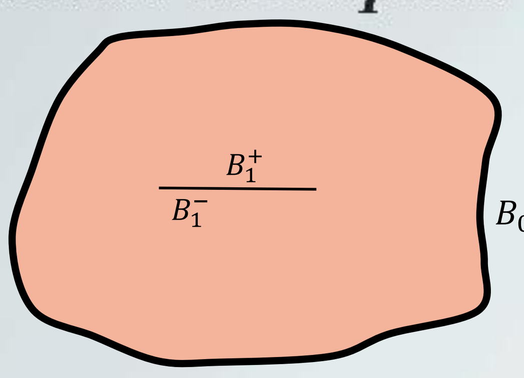
Abstract :Based on the successful experience of application the meshfree boundary integral equation method to calculating the conduction shape factor of heat exchanger tubes, this paper extends the meshfree boundary integral equation method to calculate the conduction shape factor of heat exchanger tubes containing a slit. The difference between the present method and the conventional boundary element method is that we use the local exact solution and Gaussian quadrature to technically determine the singular integral in the sense of Cauchy principal value and discretize the boundary integral equation, respectively. When using the present method to solve the problem containing a slit, we encounter the problem of degenerate boundary and twice occurrence of singular integral on the slit. To overcome above problems, we introduce the hypersingular integral equation to obtain independent equations and find a new local exact solution that can be suitable for the problem with a degenerate boundary. To avoid encountering the an certainty of jump function when using the Cartesian coordinate system to represent the local exact solution for a point on the slit, we adopt elliptical coordinates to construct the new local exact solution. After comparing the exact solutions of conduction shape factor provided in references [2] and [3], present results are consistent with those results. Finally, we also employ the conventional boundary element method (BEM) and finite element method (FEM) to calculate the temperature distribution on the cross-section of exchanger tubes. The present results make a good agreement with those results of BEM and FEM.

Problem description



Governing equation

Laplace equation : $\nabla^2 u(\mathbf{x}) = 0 , \mathbf{x} \in D$

Outer and inner boundaries are isothermal condition

$$\begin{cases} u(x) = u_1 = 400k, & x \in B_1 \\ u(x) = u_0 = 300k, & x \in B_0 \end{cases}$$

Conduction Shape Factor: S

$$S = \frac{\dot{Q}}{L \times k \times (u_1 - u_0)}$$

$\dot{Q}$ is the steady rate of heat transfer

u_1 is the temperature of inner boundary

u_0 is the temperature of outer boundary

L is the length of exchanger tube

k is the thermal conductivity

Meshfree boundary integral equation method

$$0 = \int_B T(\mathbf{s},\mathbf{x})[u(\mathbf{s}) - w(\mathbf{s})]dB(\mathbf{s}) - \int_B U(\mathbf{s},\mathbf{x})[t(\mathbf{s}) - w_p(\mathbf{s})]dB(\mathbf{s}),\ \mathbf{x} \in B$$

$$0 = \int_B M(\mathbf{s},\mathbf{x})[u(\mathbf{s}) - w(\mathbf{s})]dB(\mathbf{s}) - \int_B L(\mathbf{s},\mathbf{x})[t(\mathbf{s}) - w_p(\mathbf{s})]dB(\mathbf{s}),\ \mathbf{x} \in B$$

$$t(\mathbf{s}) = \frac{\partial u(\mathbf{s})}{\partial n(\mathbf{s})} \quad w_p(\mathbf{s}) = \frac{\partial w(\mathbf{s})}{\partial n(\mathbf{s})} \quad \begin{matrix} B = B_0 \cup B_1 \\ B_1 = B_1^+ \cup B_1^- \end{matrix}$$

For $\mathbf{x} \in B_o$ $\Rightarrow$ Ordinary boundary

$$0 = \int_B T(\mathbf{s},\mathbf{x})[u(\mathbf{s}) - w(\mathbf{s})]dB(\mathbf{s}) - \int_B U(\mathbf{s},\mathbf{x})[t(\mathbf{s}) - w_p(\mathbf{s})]dB(\mathbf{s}),\ \mathbf{x} \in B_o$$

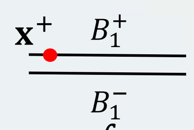
$$w(\mathbf{s}) = u(\mathbf{x}) + [(s_x - x)n_x(\mathbf{x}) + (s_y - y)n_y(\mathbf{x})]t(\mathbf{x}).$$

Gaussian integral

$$0 = \sum_{j=1}^{N_g} \omega_j T(\mathbf{s}_j, \mathbf{x}_i)[u_j - w_j^i]j(\xi_j) - \sum_{j=1}^{N_g} \omega_j U(\mathbf{s}_j, \mathbf{x}_i)[t_j - w_{p\ j}^i]j_b(\xi_j)$$

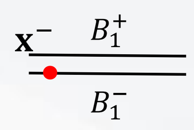
$$j_b(\xi) = \left(\frac{\tau_L - \tau_0}{2}\right)\sqrt{(x'(\tau))^2 + (y'(\tau))^2}\Big|_{\tau=\tau(\xi)}$$

For $\mathbf{x} \in B_1$ $\Rightarrow$ Degenerate boundary



$$0 = \int_{B_1} T(\mathbf{s},\mathbf{x})\,[u(\mathbf{s}) - w(\mathbf{s})]dB(\mathbf{s}) - \int_{B_1} U(\mathbf{s},\mathbf{x})\,[t(\mathbf{s}) - w_p(\mathbf{s})]dB(\mathbf{s})$$
$$+ \int_{B_0} T(\mathbf{s},\mathbf{x})\,u(\mathbf{s})dB(\mathbf{s}) - \int_{B_0} U(\mathbf{s},\mathbf{x})\,t(\mathbf{s})dB(\mathbf{s})\ ,\mathbf{x} \in B_1^+$$

$$w(\mathbf{s}) = f_1(\mathbf{s},\mathbf{x}^+)u(\mathbf{x}^+) + f_2(\mathbf{s},\mathbf{x}^+)u(\mathbf{x}^-) + g_1(\mathbf{s},\mathbf{x}^+)t(\mathbf{x}^+) + g_2(\mathbf{s},\mathbf{x}^+)t(\mathbf{x}^-)$$



$$0 = \int_{B_1} M(\mathbf{s},\mathbf{x})\,[u(\mathbf{s}) - w(\mathbf{s})]dB(\mathbf{s}) - \int_{B_1} L(\mathbf{s},\mathbf{x})\,[t(\mathbf{s}) - w_p(\mathbf{s})]dB(\mathbf{s})$$
$$+ \int_{B_0} M(\mathbf{s},\mathbf{x})\,u(\mathbf{s})dB(\mathbf{s}) - \int_{B_0} L(\mathbf{s},\mathbf{x})\,t(\mathbf{s})dB(\mathbf{s})\ ,\mathbf{x} \in B_1^-$$

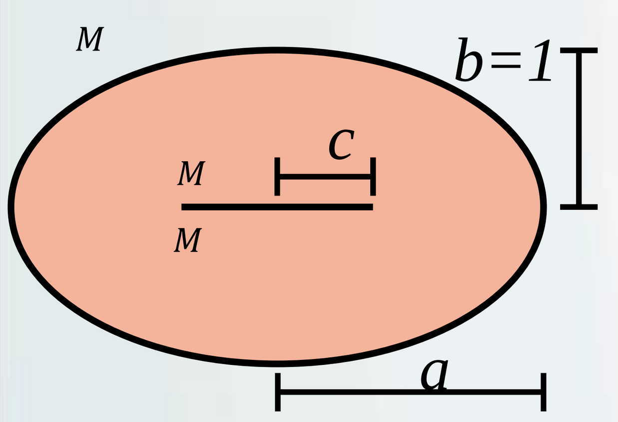
$$w(\mathbf{s}) = f_1(\mathbf{s},\mathbf{x}^-)u(\mathbf{x}^+) + f_2(\mathbf{s},\mathbf{x}^-)u(\mathbf{x}^-) + g_1(\mathbf{s},\mathbf{x}^-)t(\mathbf{x}^+) + g_2(\mathbf{s},\mathbf{x}^-)t(\mathbf{x}^-)$$

Results and discussion

Shape function of local exact solution for the problem with a degenerate boundary

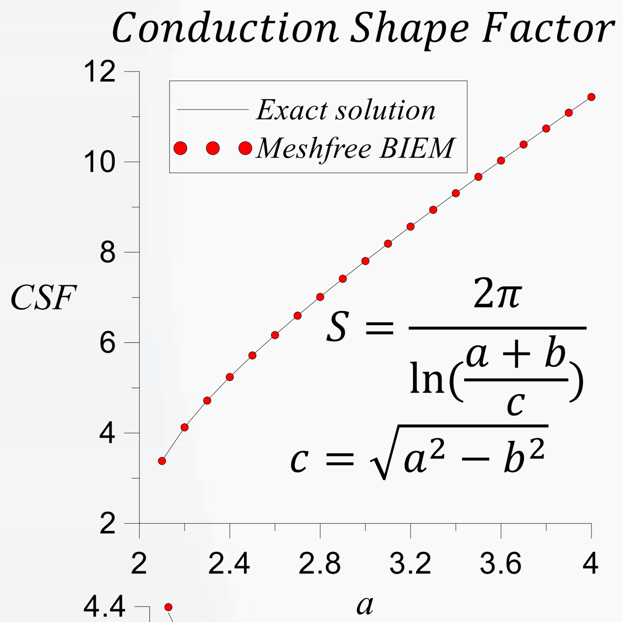
$f_1(\mathbf{s},\mathbf{x}^+) = \frac{e^{\xi_x}}{\cos(\eta_{x^+})}e^{-\xi_s}\cos(\eta_s) - \frac{1}{2}\frac{e^{2\xi_x}}{\cos(2\eta_{x^+})}e^{-2\xi_s}\cos(2\eta_s) + \frac{e^{\xi_x}}{\sin(\eta_{x^+})}e^{-\xi_s}\sin(\eta_s) - \frac{1}{2}\frac{e^{2\xi_x}}{\sin(2\eta_{x^+})}e^{-2\xi_s}\sin(2\eta_s)$	$g_1(\mathbf{s},\mathbf{x}^+) = -\frac{1}{2}J(\mathbf{x}^+)[\frac{e^{\xi_x}}{\cos(\eta_{x^+})}e^{-\xi_s}\cos(\eta_s) - \frac{e^{2\xi_x}}{\cos(2\eta_{x^+})}e^{-2\xi_s}\cos(2\eta_s) + \frac{e^{\xi_x}}{\sin(\eta_{x^+})}e^{-\xi_s}\sin(\eta_s) - \frac{e^{2\xi_x}}{\sin(2\eta_{x^+})}e^{-2\xi_s}\sin(2\eta_s)]$
$f_2(\mathbf{s},\mathbf{x}^+) = \frac{e^{\xi_x}}{\cos(\eta_{x^+})}e^{-\xi_s}\cos(\eta_s) - \frac{1}{2}\frac{e^{2\xi_x}}{\cos(2\eta_{x^+})}e^{-2\xi_s}\cos(2\eta_s) - \frac{e^{\xi_x}}{\sin(\eta_{x^+})}e^{-\xi_s}\sin(\eta_s) + \frac{1}{2}\frac{e^{2\xi_x}}{\sin(2\eta_{x^+})}e^{-2\xi_s}\sin(2\eta_s)$	$g_2(\mathbf{s},\mathbf{x}^+) = -\frac{1}{2}J(\mathbf{x}^+)[\frac{e^{\xi_x}}{\cos(\eta_{x^+})}e^{-\xi_s}\cos(\eta_s) - \frac{e^{2\xi_x}}{\cos(2\eta_{x^+})}e^{-2\xi_s}\cos(2\eta_s) - \frac{e^{\xi_x}}{\sin(\eta_{x^+})}e^{-\xi_s}\sin(\eta_s) + \frac{e^{2\xi_x}}{\sin(2\eta_{x^+})}e^{-2\xi_s}\sin(2\eta_s)]$

$f_1(\mathbf{s},\mathbf{x}^-) = \frac{e^{\xi_x}}{\cos(\eta_{x^-})}e^{-\xi_s}\cos(\eta_s) - \frac{1}{2}\frac{e^{2\xi_x}}{\cos(2\eta_{x^-})}e^{-2\xi_s}\cos(2\eta_s) + \frac{e^{\xi_x}}{\sin(-\eta_{x^-})}e^{-\xi_s}\sin(\eta_s) - \frac{1}{2}\frac{e^{2\xi_x}}{\sin(-2\eta_{x^-})}e^{-2\xi_s}\sin(2\eta_s)$	$g_1(\mathbf{s},\mathbf{x}^-) = -\frac{1}{2}J(\mathbf{x}^-)[\frac{e^{\xi_x}}{\cos(\eta_{x^-})}e^{-\xi_s}\cos(\eta_s) - \frac{e^{2\xi_x}}{\cos(2\eta_{x^-})}e^{-2\xi_s}\cos(2\eta_s) + \frac{e^{\xi_x}}{\sin(-\eta_{x^-})}e^{-\xi_s}\sin(\eta_s) - \frac{e^{2\xi_x}}{\sin(-2\eta_{x^-})}e^{-2\xi_s}\sin(2\eta_s)]$
$f_2(\mathbf{s},\mathbf{x}^-) = \frac{e^{\xi_x}}{\cos(\eta_{x^-})}e^{-\xi_s}\cos(\eta_s) - \frac{1}{2}\frac{e^{2\xi_x}}{\cos(2\eta_{x^-})}e^{-2\xi_s}\cos(2\eta_s) - \frac{e^{\xi_x}}{\sin(-\eta_{x^-})}e^{-\xi_s}\sin(\eta_s) + \frac{1}{2}\frac{e^{2\xi_x}}{\sin(-2\eta_{x^-})}e^{-2\xi_s}\sin(2\eta_s)$	$g_2(\mathbf{s},\mathbf{x}^-) = -\frac{1}{2}J(\mathbf{x}^-)[\frac{e^{\xi_x}}{\cos(\eta_{x^-})}e^{-\xi_s}\cos(\eta_s) - \frac{e^{2\xi_x}}{\cos(2\eta_{x^-})}e^{-2\xi_s}\cos(2\eta_s) - \frac{e^{\xi_x}}{\sin(-\eta_{x^-})}e^{-\xi_s}\sin(\eta_s) + \frac{e^{2\xi_x}}{\sin(-2\eta_{x^-})}e^{-2\xi_s}\sin(2\eta_s)]$

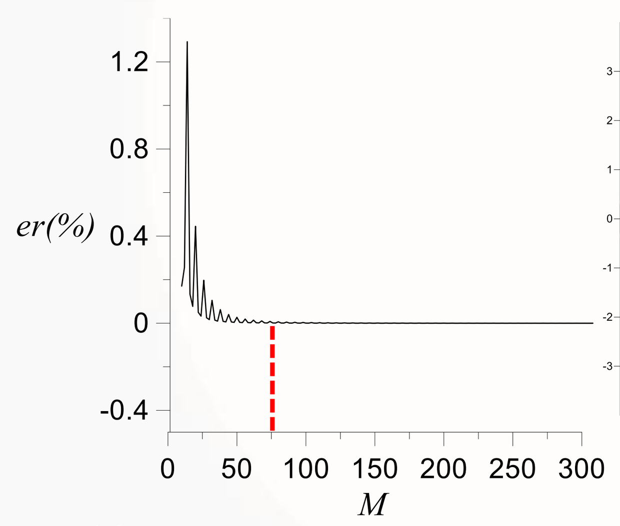


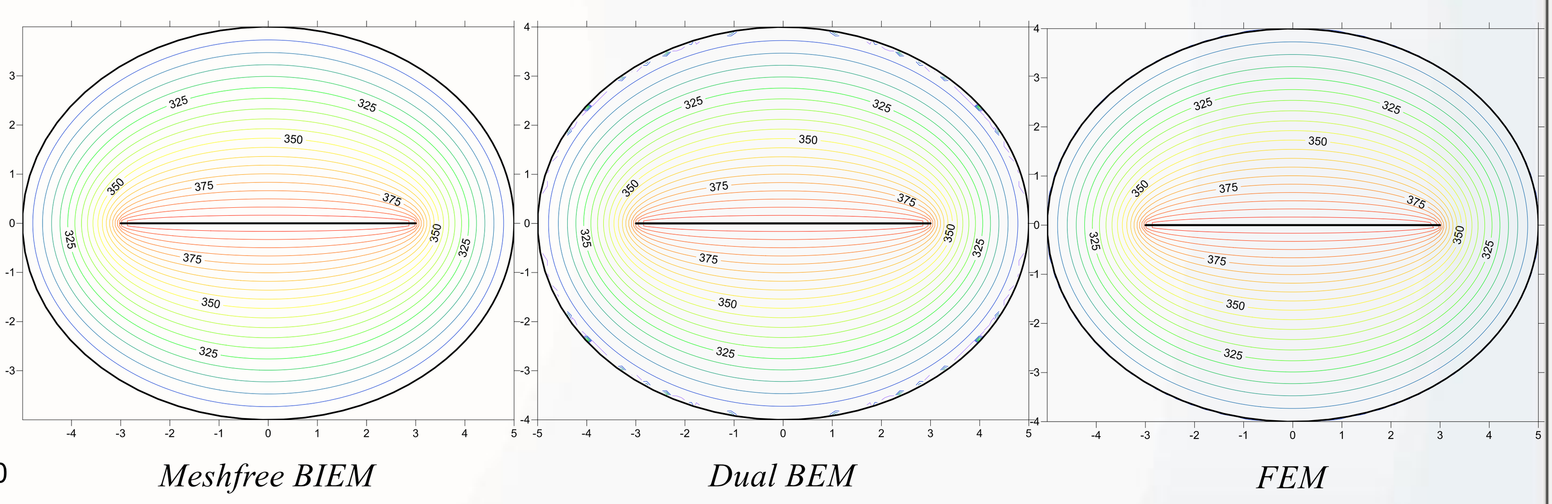
Elliptical tube with a focal slit

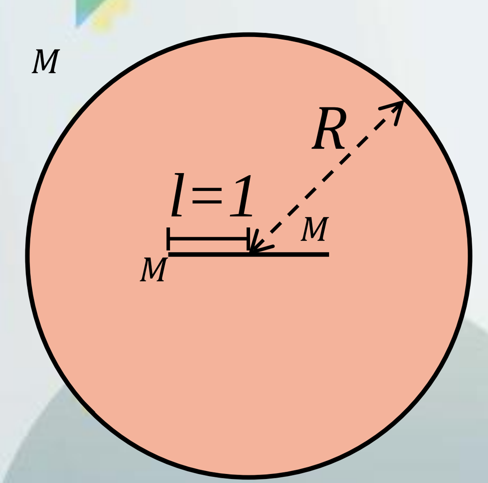
Conduction Shape Factor



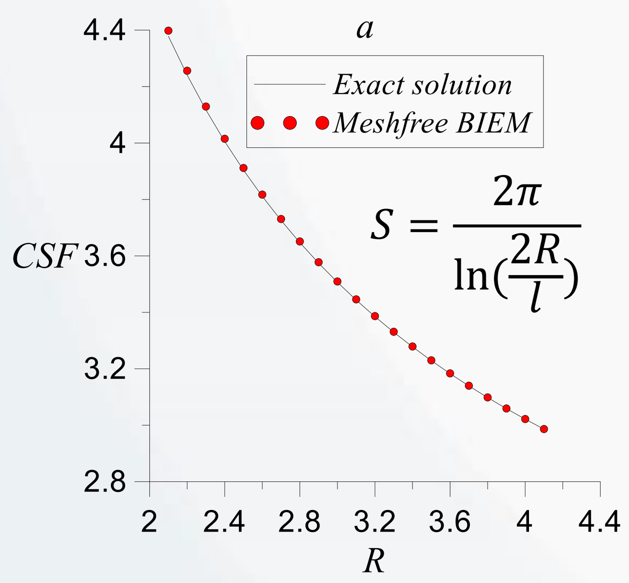
Relative error

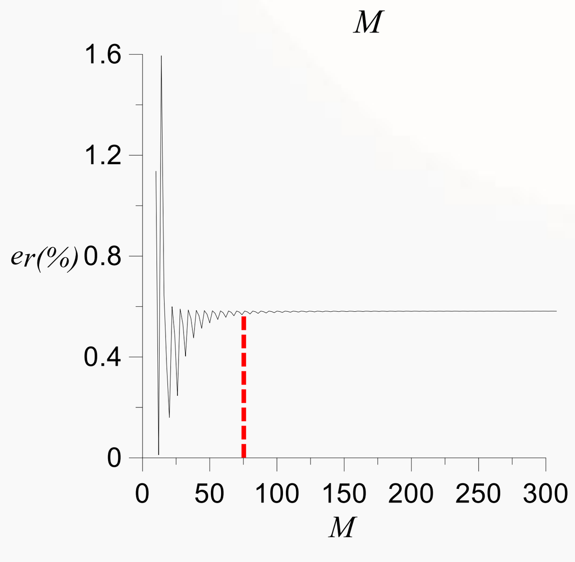


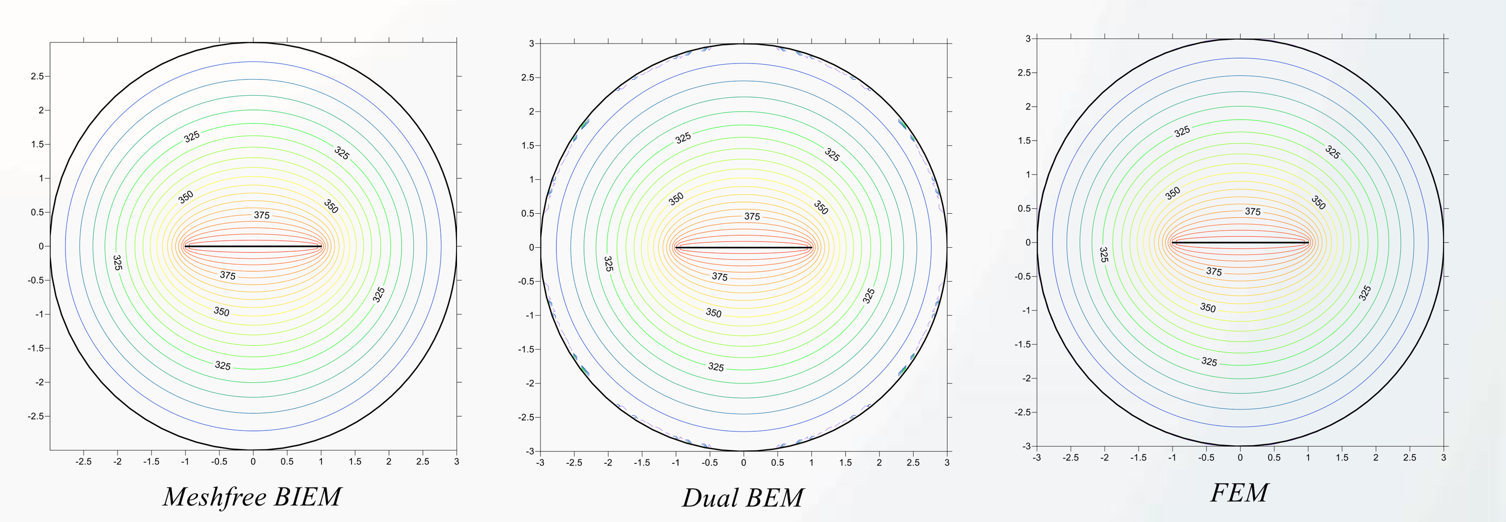




Circular tube with a central slit







Conclusions

1. The conduction shape factor of exchanger tubes with a slit can be successfully calculated by using the meshfree boundary integral equation method .
2. To overcome the issue of degenerate boundaries, it is necessary to use the hypersingular integral equation.
3. The appropriate local exact solution for a problem with a degenerate boundary can be constructed by using the elliptical coordinates.

References

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