2023

【海報論文】

□一般組 ■研究生組

□大學組

11th 台灣TAIWAN 工業與應用數學會年會

TWSIAM Annual Meeting

Finite element analysis on yield surfaces of trabecular bone

Hong-Yi Wu (吳鴻毅), Department of Engineering Science, National Cheng Kung University, Tainan 70101, Taiwan

Advisor: Prof. Li-Wei Liu (劉立偉), Department of Civil Engineering, National Taiwan University, Taipei 10617, Taiwan

Abstract

Trabecular bone has cellular microstructure and behaves the asymmetry yielding in tension and compression. In our study, we adopted the Voronoi honeycomb to be the microstructure of the trabecular bone and used the generalized Hill yield function in finite element analysis. To study the yield surface evolution of trabecular bone, the mesh size, the cell number and the size of the representative block in finite element analysis is investigated and selected based on the analysis of the mesh convergence, the boundary effect, and the size effect. We designed the probing paths and pre-loading paths to detect the initial yield surface and subsequent yield surfaces with different pre-loading paths. According to the observation of the yield surface evolution, the Bauschinger effect and the hardening behavior (isotropic, kinematic, rotation, distortional) of the trabecular bone can be recognized. The finite element simulation shows that the variation of yield surface area and the aspect ratio under different pre-loading paths. In addition, the Bauschinger effect appears under certain monotonic pre-loading paths.

Introduction

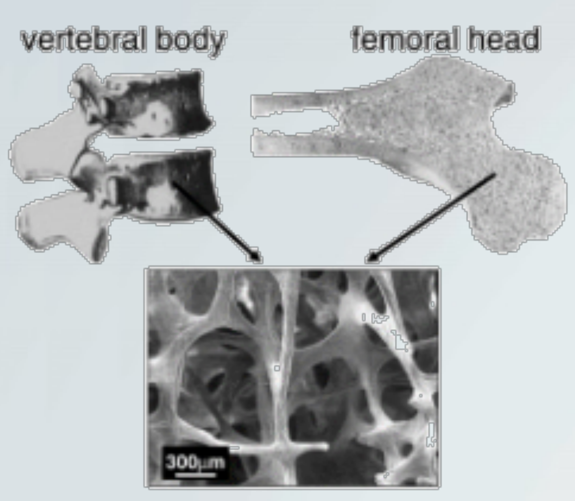


Fig. 1: Certain bones (or parts of bones), such as the vertebra or the femoral head, are filled with spongy structure called "trabecular bone". [1]

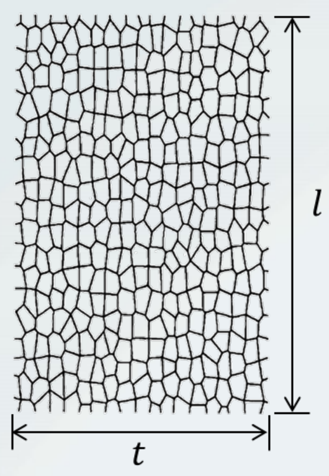


Fig. 2: Voronoi diagram of a model of vertebral trabecular bone, note that l: t = 3.5 : 2.33 . [2]

	Young's modulus (MPa)	Tangent modulus (MPa)	Yield stress (MPa)
Compression	350	0	2.02
Tension			1.72
Shear			0.9

Table 1: Material constants of vertebra trabecular bone. [3] [4]

Problem description

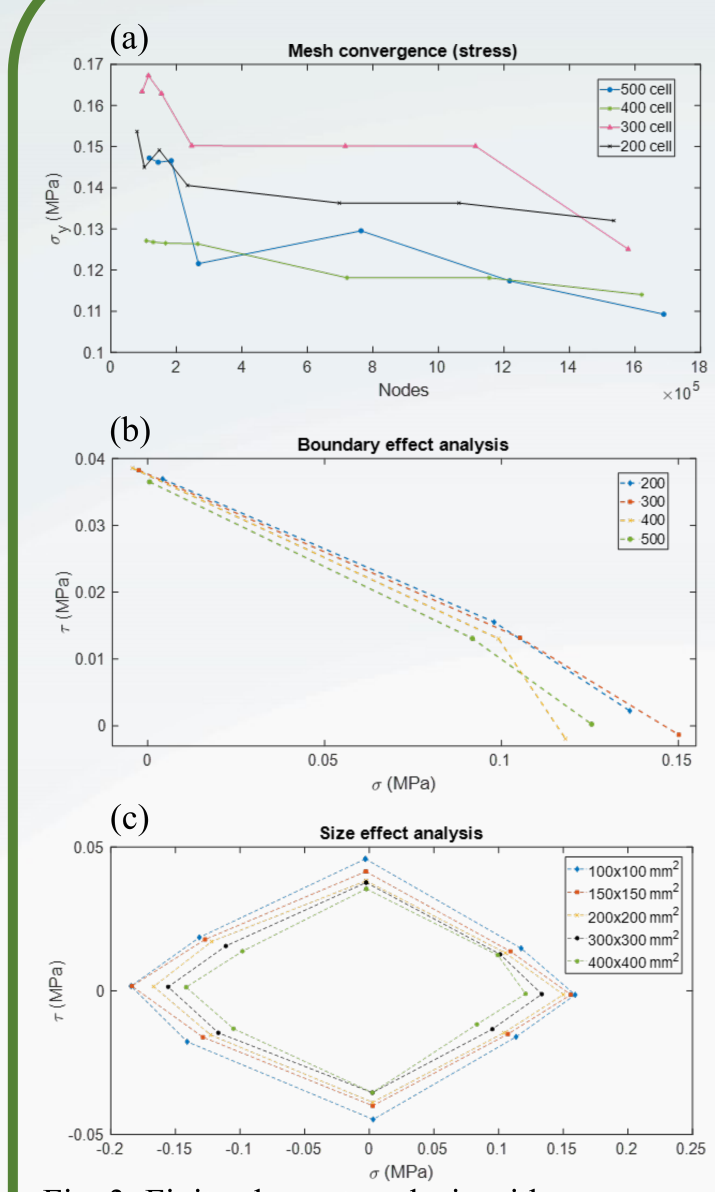


Fig. 3: Finite element analysis with
(a) mesh convergence of different cell counts,
(b) boundary effect analysis of different cell counts and
(c) size effect of different representative block area (cell counts = 300).

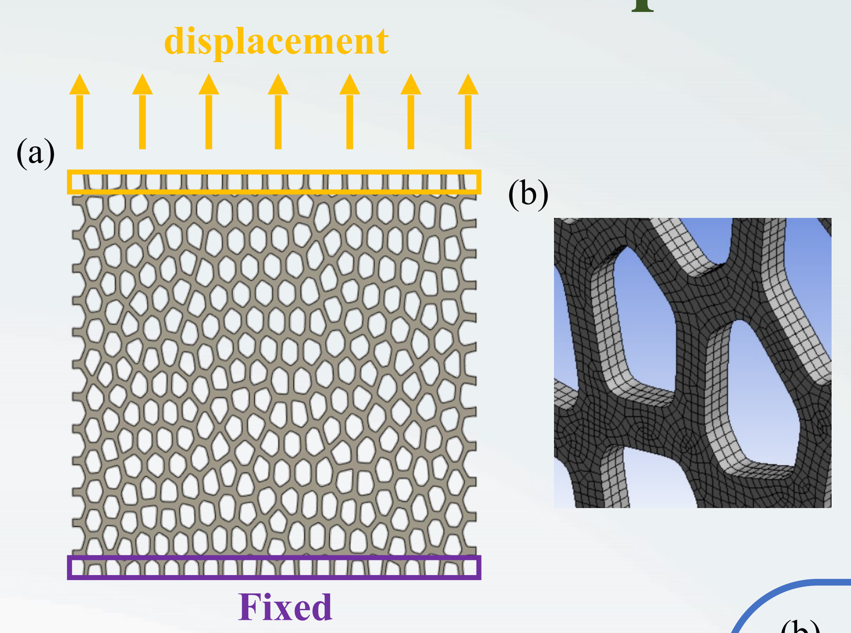


Fig. 4:
(a) Voronoi diagram of trabecular bone (area= 300 x 300 mm²).
(b) Mesh in thickness direction.

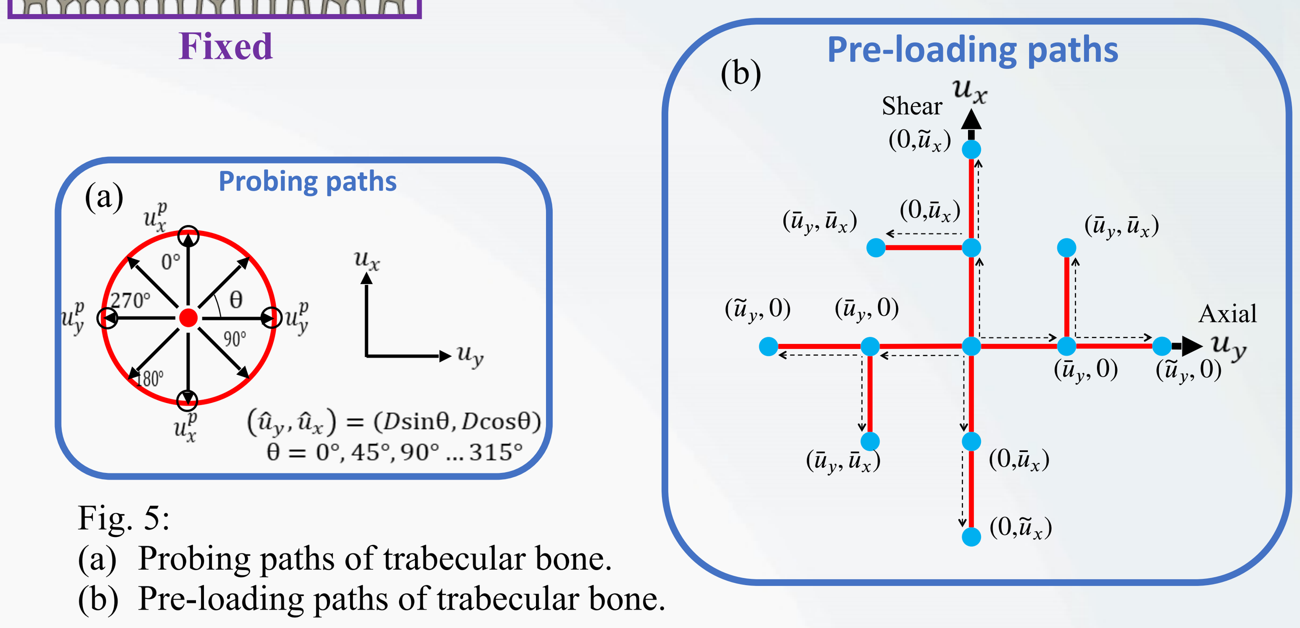


Fig. 5:
(a) Probing paths of trabecular bone.
(b) Pre-loading paths of trabecular bone.

Results

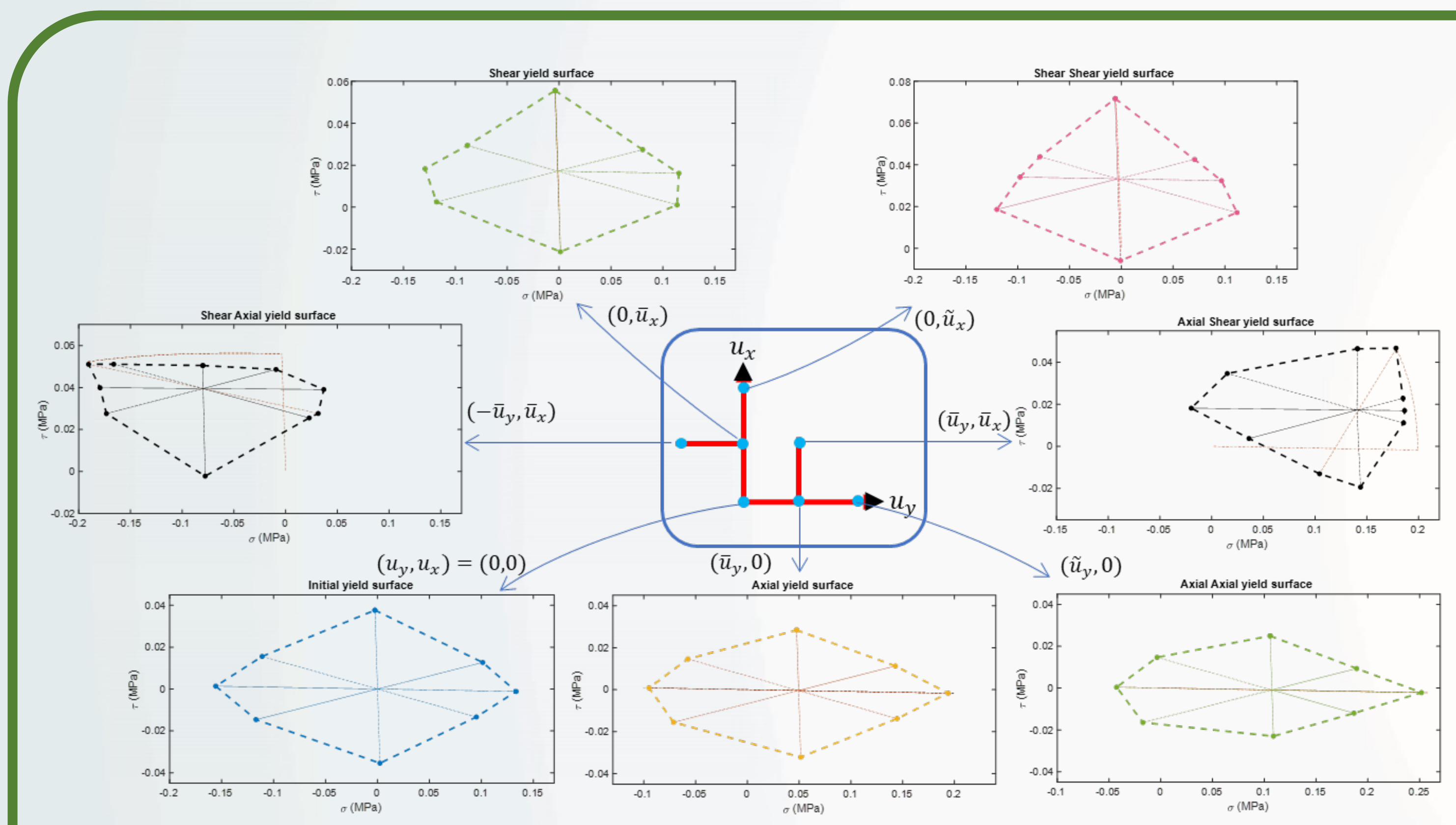


Fig. 6: Yield surface evolution of "axial-tension and shear" pre-loading paths.

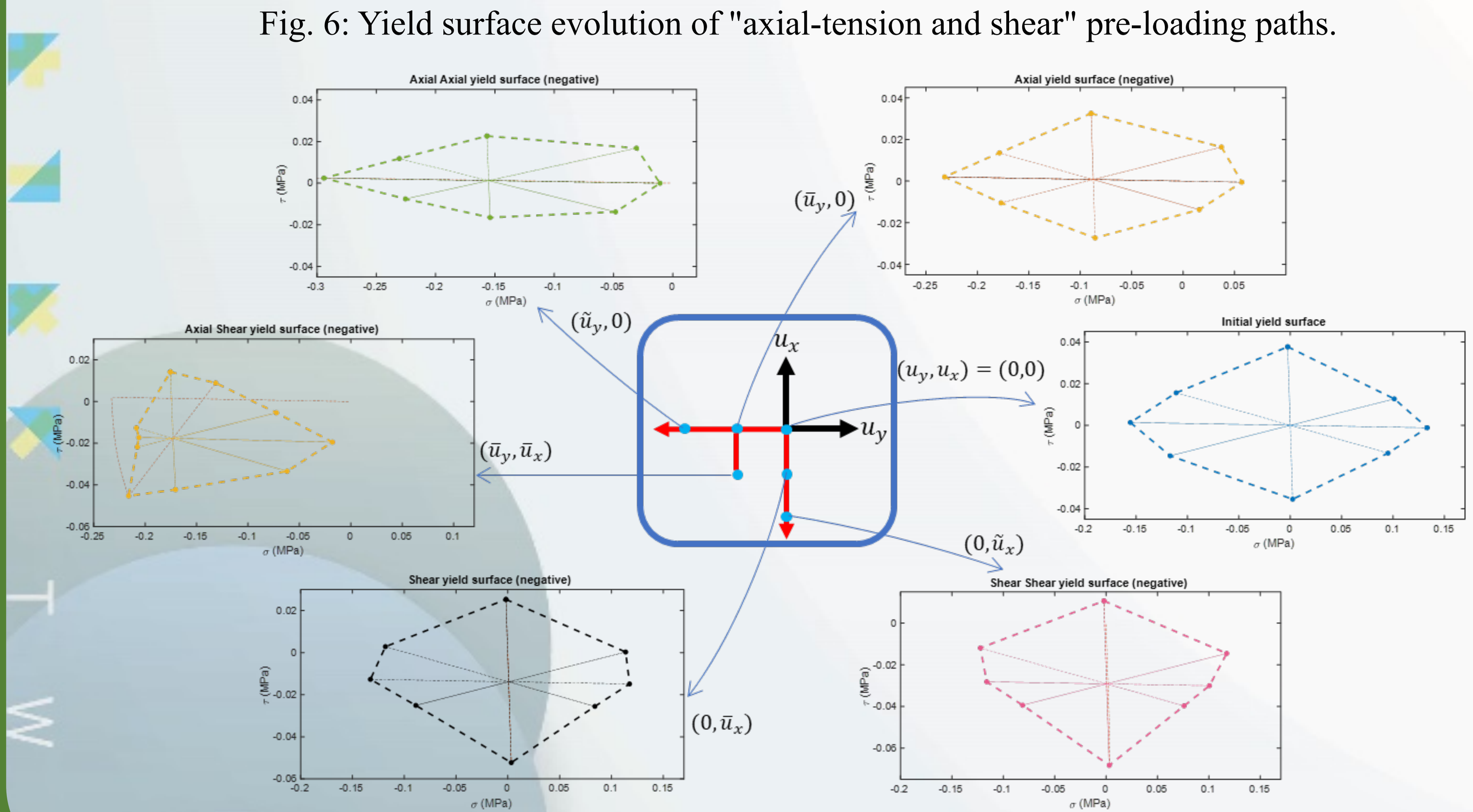


Fig. 7: Yield surface evolution of "axial-compression and shear" pre-loading paths.

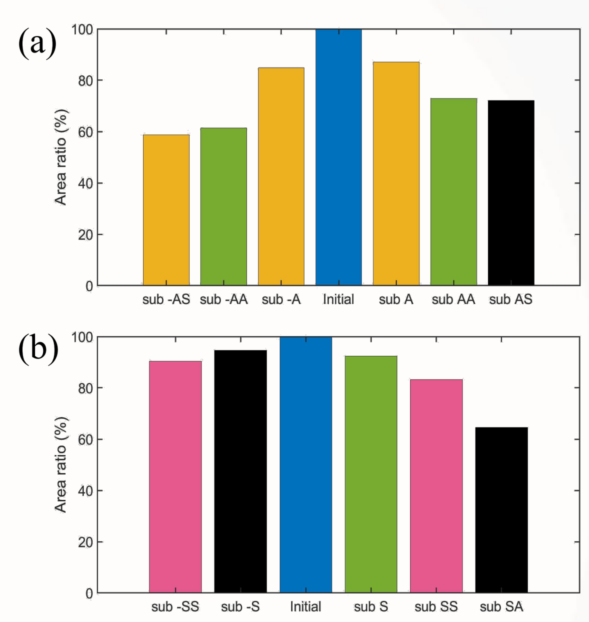


Fig. 8: Area ratio of (a) axial first and (b) shear first pre-loadings.

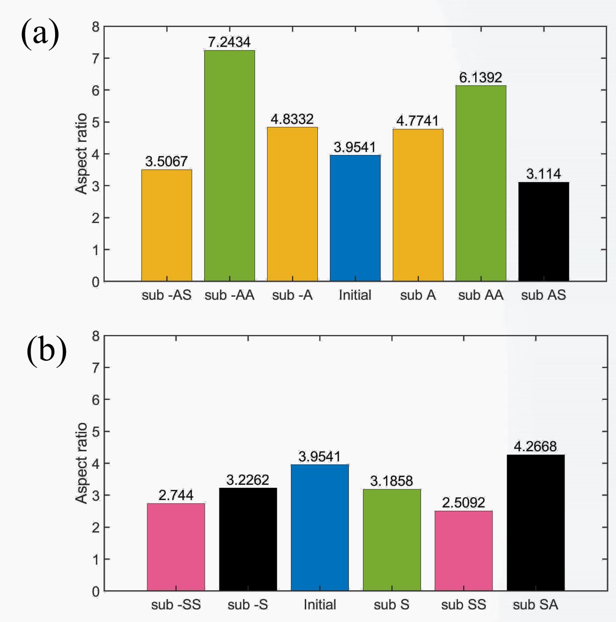
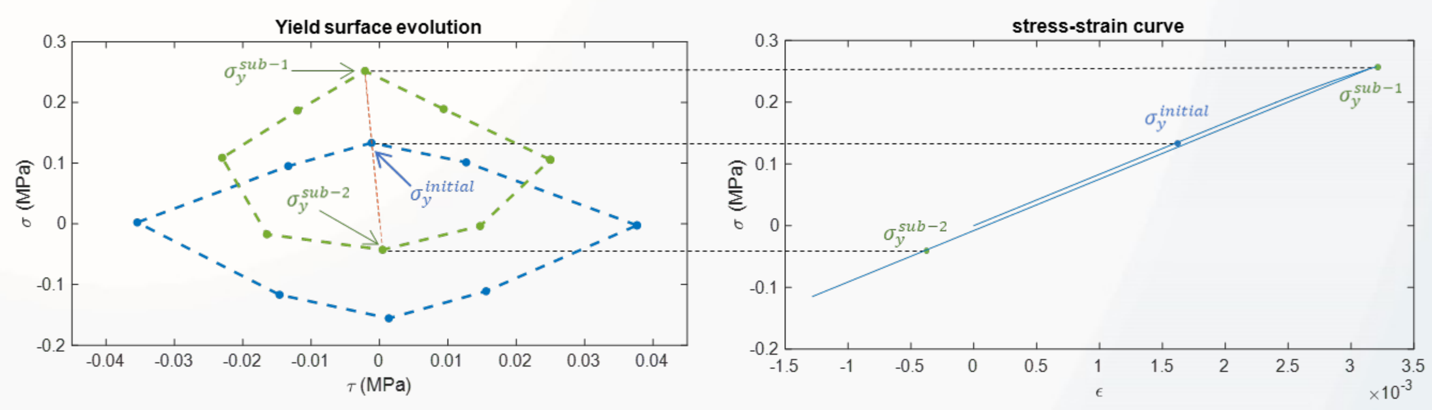


Fig. 9: Aspect ratio of (a) axial first and (b) shear first pre-loadings.



Bauschinger effect: $\Delta\sigma_y^e = (\sigma_y^{sub-1} - \sigma_y^{sub-2})$ in range $(2\sigma_y^{initial}, 2\sigma_y^{sub-1})$

Fig. 10: Yield surfaces and stress-strain curve to analysis Bauschinger effect.

(a)	Bauschinger effect
pre-loading A-path	√
pre-loading AA-path	√
pre-loading -A-path	x
pre-loading -AA-path	x

(b)	Bauschinger effect
pre-loading S-path	√
pre-loading SS-path	√
pre-loading -S-path	√
pre-loading -SS-path	√

Table 2: Bauschinger effect determination with (a) axial first and (b) shear first yield surface evolution.

Conclusion

We established the Voronoi diagram with 300 cells and 300 mm² area for trabecular bone and modeled it by using the generalized Hill yield function and referring the material constants of vertebra part. The finite element analysis shows same tendency for variation of yield surface area under axial-tension and axial-compression pre-loading paths and exhibits the minor area decrease of yield surface under positive and negative shear pre-loading paths. On the other hand, the aspect ratio of the yield surface increases when the axial pre-loading paths are applied but opposite trend is observed for the yield surface under shear-first pre-loading paths. Furthermore, the Bauschinger effect appears in the monotonic pre-loadings except two axial-compression pre-loading paths.

Reference

[1] P. Fratzl, R. Weinkamer: Nature's hierarchical materials, Progress in Materials Science, Vol. 52, 2007, pp. 1262-1334.
[2] M. J. Silva and L. J. Gibson: Modeling the Mechanical Behavior of Vertebral Trabecular Bone: Effects of Age-Related Changes in Microstructure, Bone, Vol. 21, 1997, pp. 191-199.
[3] E. F. Morgan and T. M. Keaveny: Dependence of yield strain of human trabecularbone on anatomic site, Journal of Biomechanics, Vol. 34, 2001, pp. 569-577.
[4] A. Sanyal, A. Gupta, H. H. Bayraktar, R. Y. Kwon and T. M. Keaveny: Shear strength behavior of human trabecular bone, Journal of Biomechanics, Vol. 45, 2012, pp. 2513-2519.
[5] L.-W. Liu, C.-Y. Yang and H.-G. Chen: Finite element analysis on yield surface evolution of cellular materials, International Journal of Mechanical Sciences, Vol. 246, 2023,108123.