2023 【海報論文】 □一般組 ■研究生組 □大學組

11th 台灣 TAIWAN 工業與應用數學會年會 TWSIAM Annual Meeting

A Phase Field Material Point Method for Modeling Fracture and Fragmentation Problems

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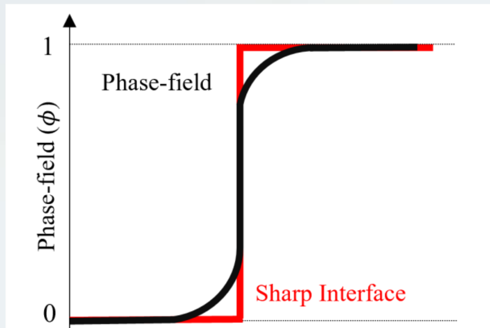
Abstract

Fragmentation problems in solid mechanics is challenging for conventional mesh-based formulations such as the finite element method (FEM). To address this issue, a phase-field material point method (PF-MPM) is presented. The phase field method regularizes the strongly discontinuous crack geometry into a diffusive damage representation, providing discretization independence. The material point method can effectively bypass mesh distortion and entanglement. A dynamic hyperbolic phase-field equation is proposed for explicit time integration. Numerical examples are provided to benchmark the performance.

Phase-Field Theory for Fracture Mechanics

- The phase-field method [1] models a sharp crack into diffusive crack (damage) represented by the phase-field (scalar) variable $\phi \in [0,1]$:

Undamaged State: $\phi = 0$ Damaged State: $\phi = 1$



Sharp and diffusive interface

- Fracture energy approximation at the crack surface:

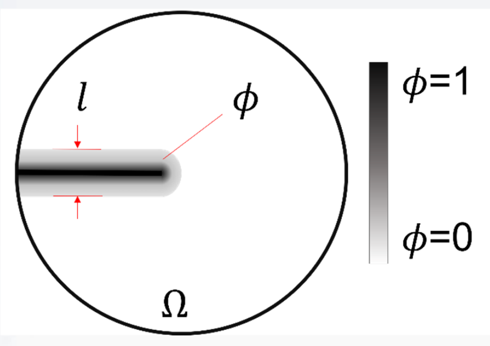
$$\int_{\Gamma_c} G_c d\Gamma \approx \int_\Omega G_c \Lambda(l,\phi) d\Omega = \frac{G_c}{2l}\int_\Omega \left(\alpha(\phi) + l^2 (\nabla\phi)^2\right) d\Omega$$

- Undamaged tensile strain energy:

$$\psi_0^+(\boldsymbol{\varepsilon}) = \frac{1}{2}\lambda\langle tr\boldsymbol{\varepsilon}\rangle + \mu tr[(\boldsymbol{\varepsilon}^+)^2]$$

- History field variable:

$$H^+ = \max_{\tau\in(0,T)} \psi_0^+(\boldsymbol{\varepsilon}(\boldsymbol{u},\tau))$$



Diffusive crack topology

- Total potential energy of the solid (Ψ):

$$\Psi(\boldsymbol{u},\phi) = \underbrace{\int_\Omega g(\phi)\psi_0^+(\boldsymbol{\varepsilon}(\boldsymbol{u}))d\Omega}_{\text{Effective strain energy}} + \underbrace{\frac{G_c}{2l}\int_\Omega \left(\alpha(\phi) + l^2(\nabla\phi)^2\right)d\Omega}_{\text{Fracture energy}}$$

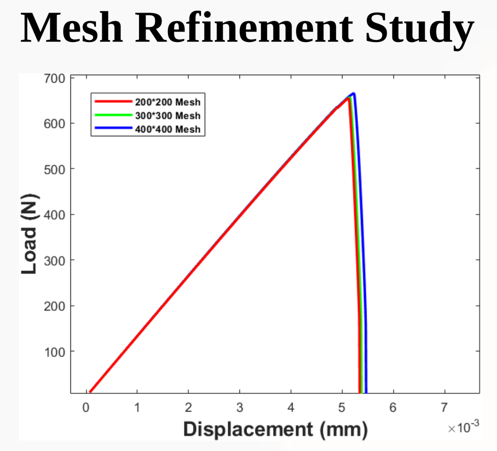
$g(\phi)$: Degradation function
l: Length scale parameter
G_c: Critical energy release rate

- Coupled equations for displacement and phase-field variable:

$$\int_\Omega \left(1-\phi^h\right)^2 \boldsymbol{\sigma}(\boldsymbol{u}^h) : \boldsymbol{\varepsilon}(\delta\boldsymbol{u}^h) d\Omega = 0$$

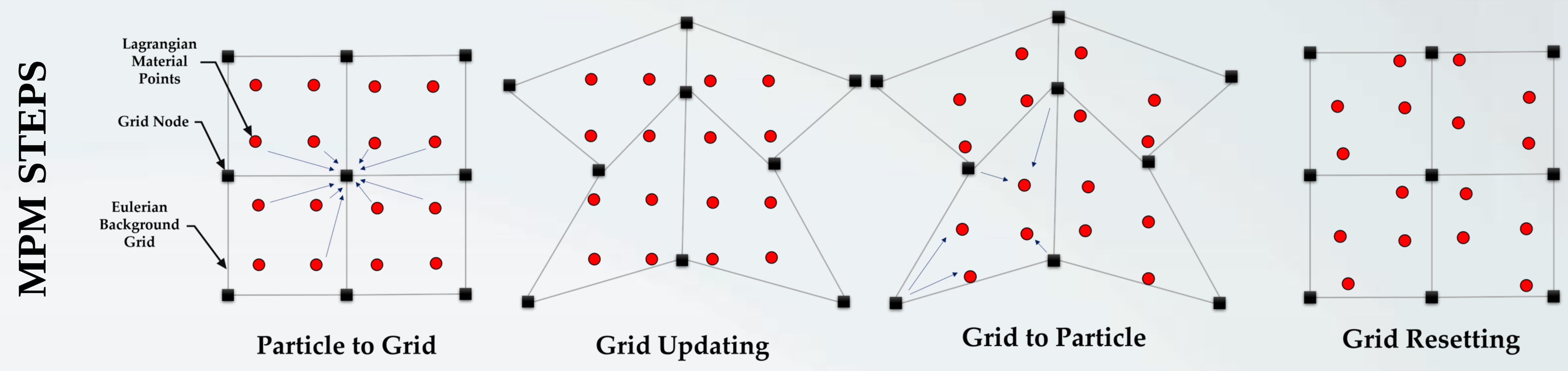
$$\int_\Omega \left(G_c l \nabla\delta\phi^h \cdot \nabla\phi^h + \left(\frac{G_c}{2l} + 2H^+\right)\phi^h\delta\phi^h - 2H^+\delta\phi^h \right) d\Omega = 0$$

Mesh Refinement Study



Dynamic Phase-Field Material Point Method

- MPM by Sulsky [2] is an efficient mesh free method to solve for fragmentation problems:



- Galerkin form of momentum balance and phase field evolution equations reads as [3]:

$$\int_\Omega \rho\delta\boldsymbol{u}^h \cdot \ddot{\boldsymbol{u}}^h d\Omega + \int_\Omega (1-\phi)^2 \nabla\delta\boldsymbol{u}^h : \boldsymbol{\sigma} d\Omega = \int_\Omega \rho\delta\boldsymbol{u}^h \cdot \boldsymbol{b} d\Omega + \int_{\partial\Omega_t} \rho\delta\boldsymbol{u}^h \cdot \boldsymbol{t}^s d\Gamma$$

$$\left(2H + \frac{G_c}{2l}\right)\int_\Omega \phi^h\delta\phi^h d\Omega - 2H\int_\Omega \delta\phi^h d\Omega + 2G_c l \int_\Omega \nabla\delta\phi^h \cdot \nabla\phi^h d\Omega = 0$$

- Hyperbolic phase-field evolution PDE can be given as [4]:

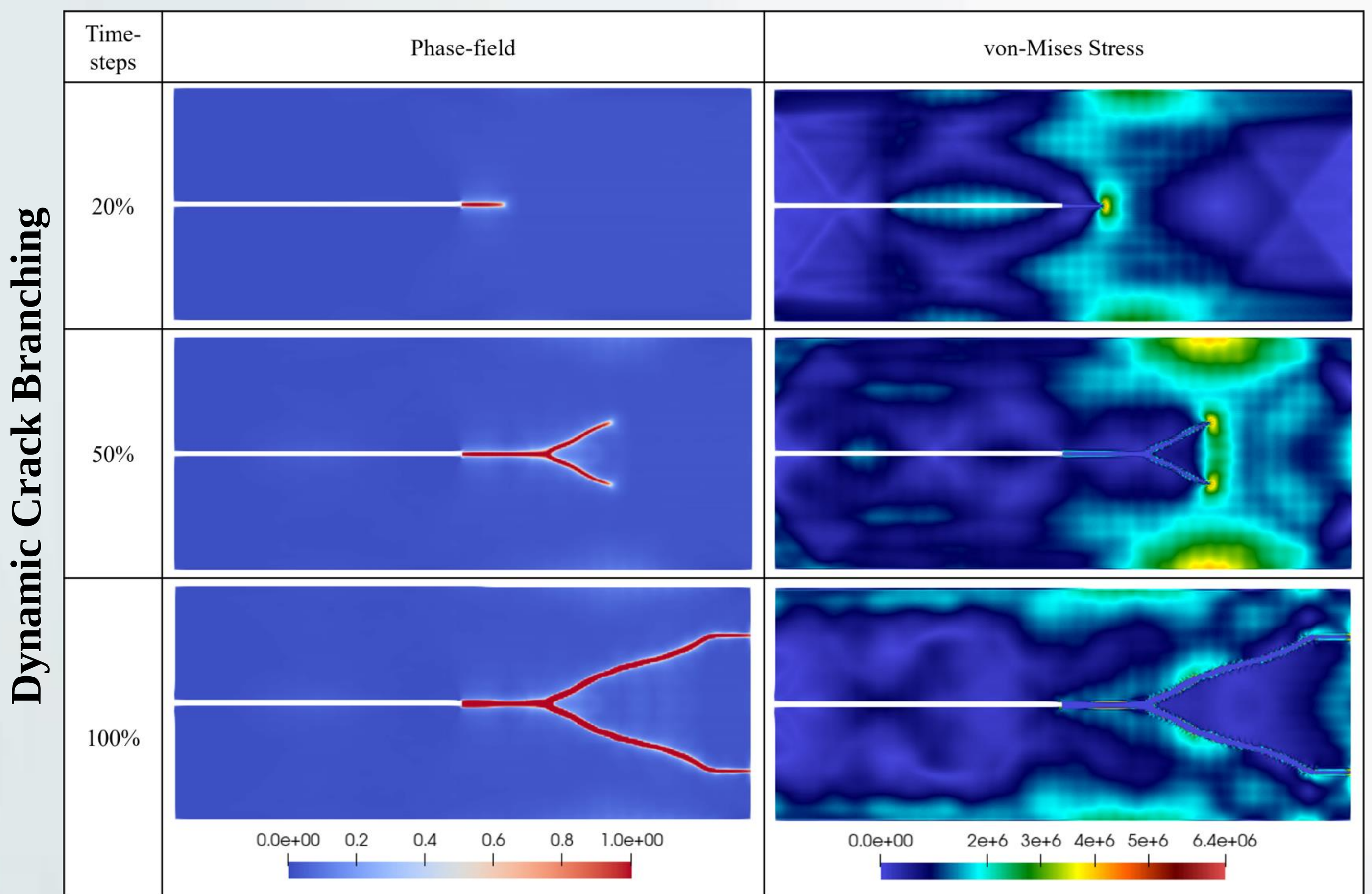
$$\frac{2G_c l}{c^2}\int_\Omega \ddot{\phi}^h\delta\phi^h d\Omega + \frac{1}{m}\int_\Omega \dot{\phi}^h\delta\phi^h d\Omega + \left(2H + \frac{G_c}{2l}\right)\int_\Omega \phi^h\delta\phi^h d\Omega - 2H\int_\Omega \delta\phi^h d\Omega + 2G_c l\int_\Omega \nabla\delta\phi^h \cdot \nabla\phi^h d\Omega = 0$$

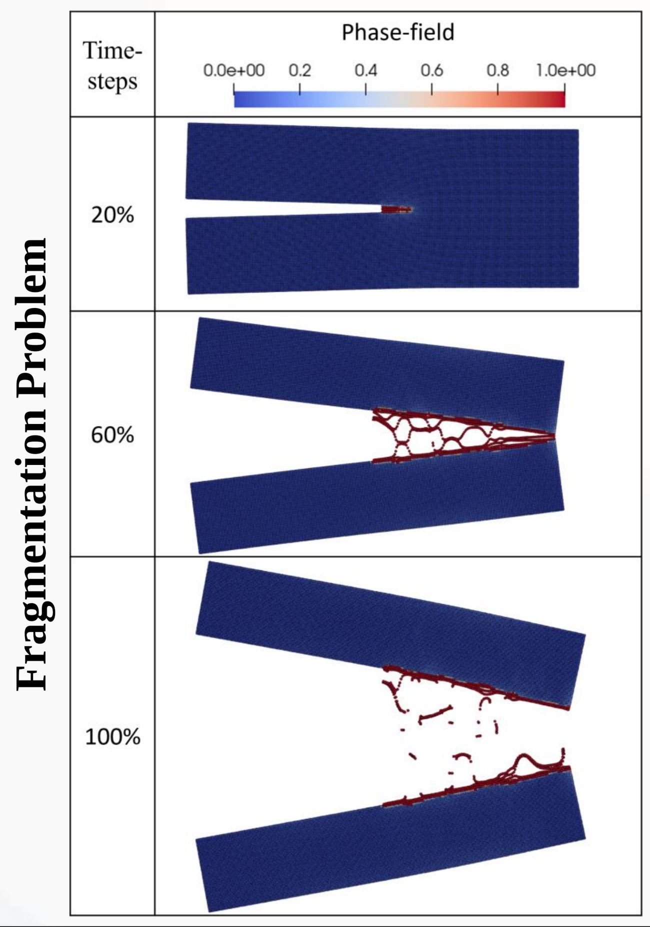
c: wave speed
m: mobility parameter

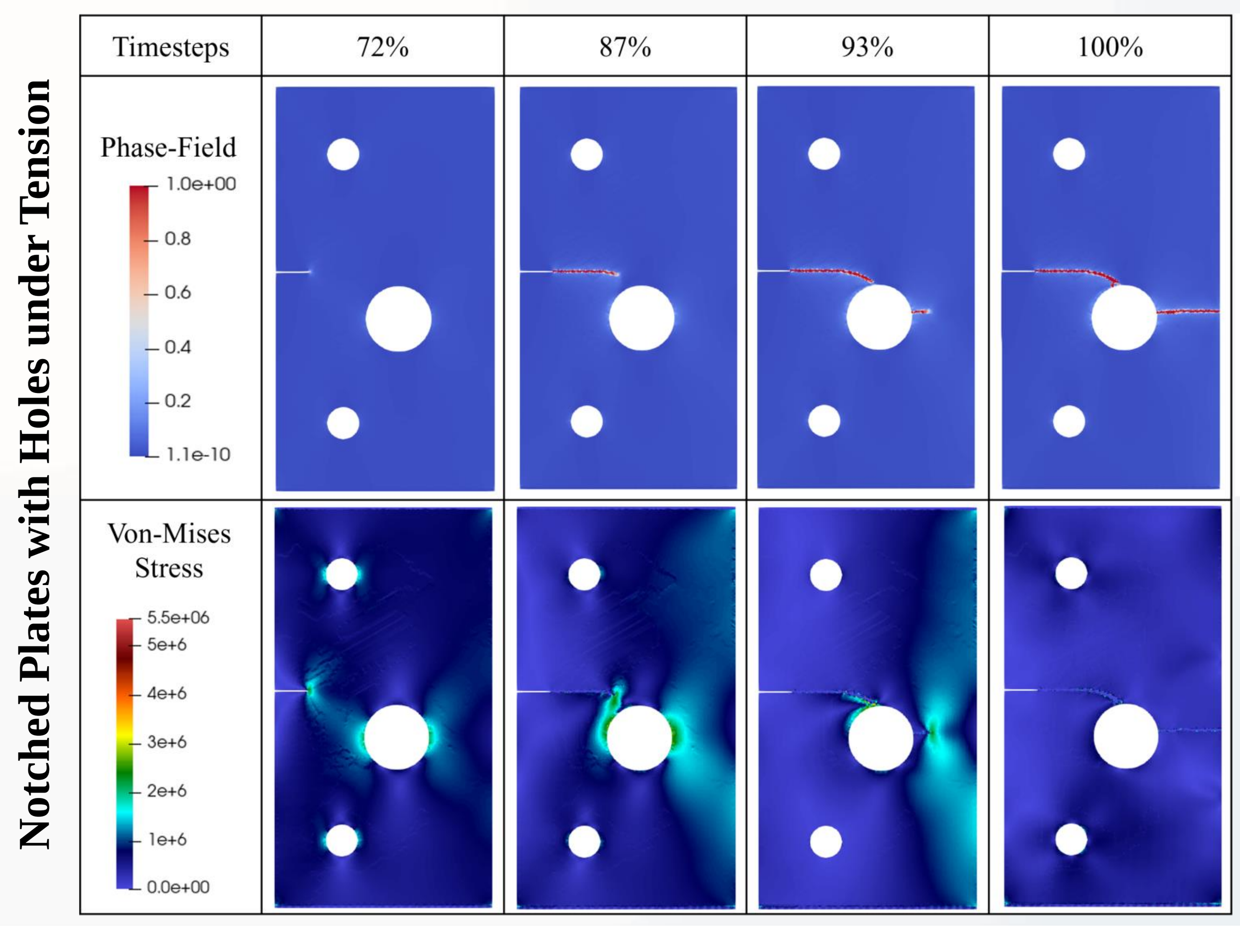
CFL Conditions: For Hyperbolic PDE: $c\Delta t \leq \Delta x$ For Parabolic PDE: $c\Delta t \leq \Delta x^2$

- CFL condition for Hyperbolic PDE allows relaxed timestep selection.

Dynamic Crack Branching, Initiation and Fragmentation Problems using PF-MPM







Remarks

- ✓ Crack initiation in complex geometry
- ✓ Crack branching
- ✓ Fragmentation of the material

Conclusion

In this study, we have presented a phase-field material point method for modeling fracture and fragmentation problems. Phase field formulation for fracture mechanics exhibits mesh independency and accurate crack propagation. By adding a second-order time derivative term in the PF-MPM, the system is converted into a hyperbolic equation that is suitable for explicit time integration, significantly reducing computational cost. In the numerical testing, PF-MPM have shown promising feature in solving fragmentation problems, that was challenging for traditional mesh-based methods like FEM. We also demonstrated the effectiveness of the dynamic phase-field formulation in simulating complex fracture behavior such as crack initiation and branching.

Reference

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