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【海報論文】

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Analytical solutions and FEM results for the support motion of a finite bar with an external viscously damped and spring boundary

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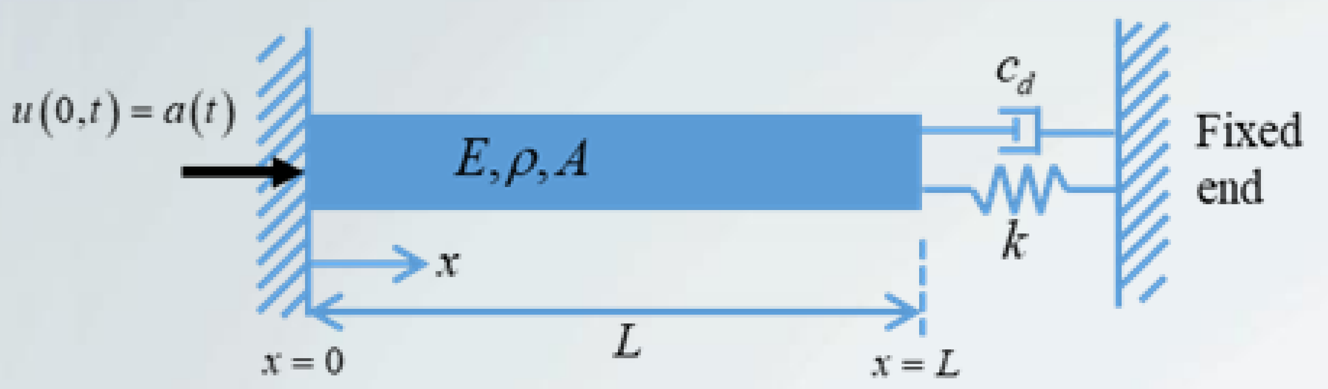
Abstract

Following the previous experience for the bar with a boundary spring or a viscous damper, we extend to solve the vibration problem of a finite bar with a viscously damped boundary in conjunction with a spring on the same side subject to the support motion on the other side. To avoid the computation of complex-valued mode and its difficulty of orthogonal condition for the continuous system, we employ two alternatives. One is the analytical method by using the idea of diamond rule based on the method of characteristics. The other numerical method, finite element method (FEM), is easily incorporated into a general program to solve this problem. The solution obtained by using the idea of diamond rule is compared with that of the FEM. Two special cases, only spring and damper alone, are also considered by using the FEM. It is found that the FEM results are in good agreement of that obtained by using the analytical approach. It is interesting that the silent area is also captured in the displacement profile by using the FEM. Slope discontinuity occurring at the time-space location of characteristic line is observed for the three cases.

Problem description



Sketch of a finite bar with a viscous damper and a spring at the end of $x = L$, subjected to a support motion at $x = 0$

Governing equation:

$$c^2\frac{\partial^2 u(x,t)}{\partial x^2}=\frac{\partial^2 u(x,t)}{\partial t^2},\quad 0<x<L,\ t>0$$

Boundary condition:

$$u(0,t)=a(t)$$

$$EA\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=L}=-c_d\left.\frac{\partial u(x,t)}{\partial t}\right|_{x=L}-ku(L,t)$$

Initial condition:

$$u(x,t)\big|_{t=0}=\phi(x)=0$$

$$\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=\varphi(x)=0.$$

Methods of solution

Method of the diamond rule

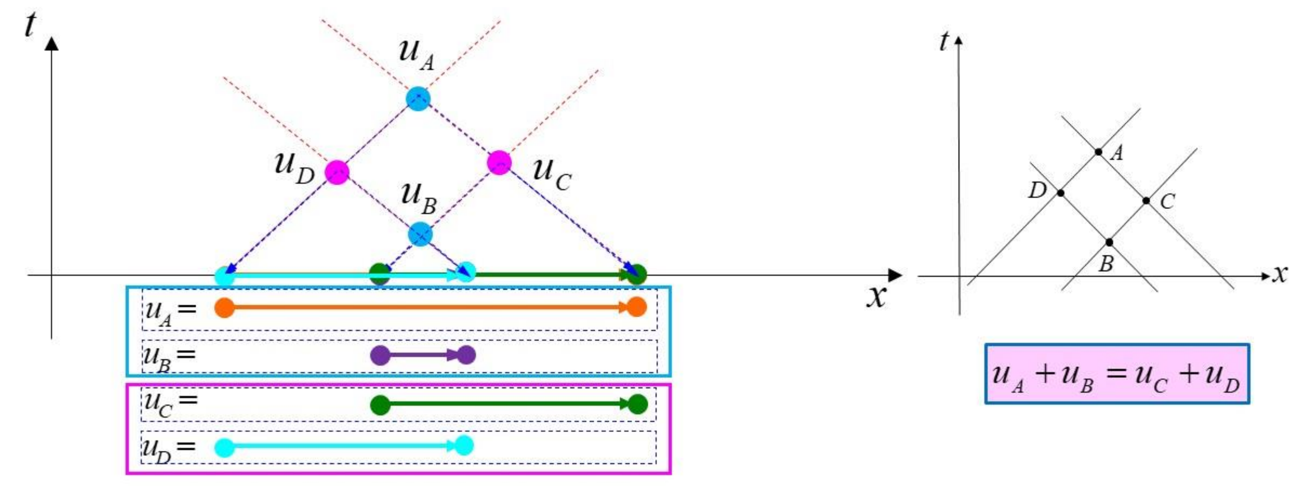
Thanks to Prof. SS Lin (林紹雄教授) and Prof. MK Kuo (郭茂坤教授) for providing information

D'Alembert solution:

$$u(x,t)=\frac{1}{2}\{\phi(x+ct)+\phi(x-ct)\}+\frac{1}{2c}\int_{x-ct}^{x+ct}\varphi(\tau)d\tau$$

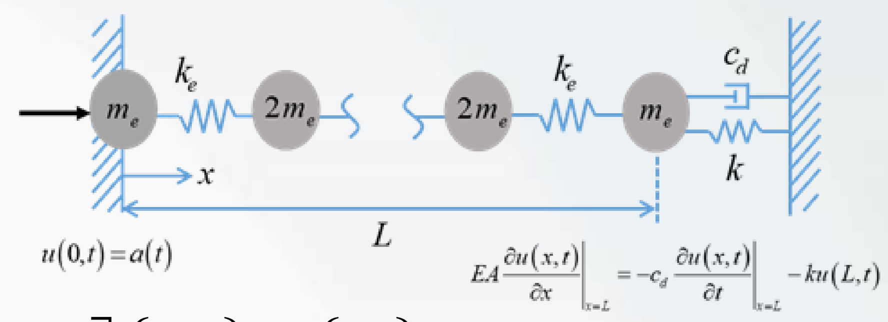
I.C.

$$\begin{cases} u(x,0)=\phi(x) \\ \left.\dfrac{\partial u(x,t)}{\partial t}\right|_{t=0}=\varphi(x)\end{cases}$$



$$u_A+u_B=u_C+u_D$$

Finite element method



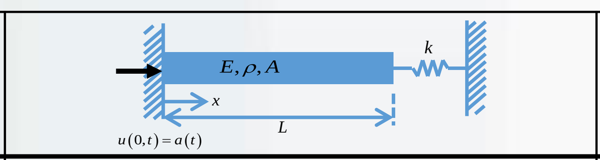
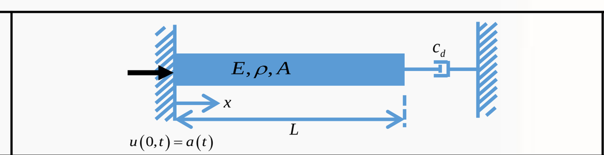
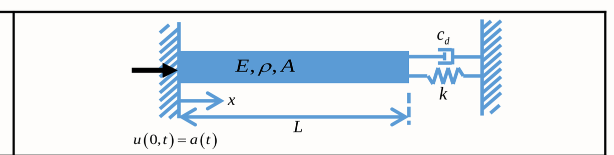
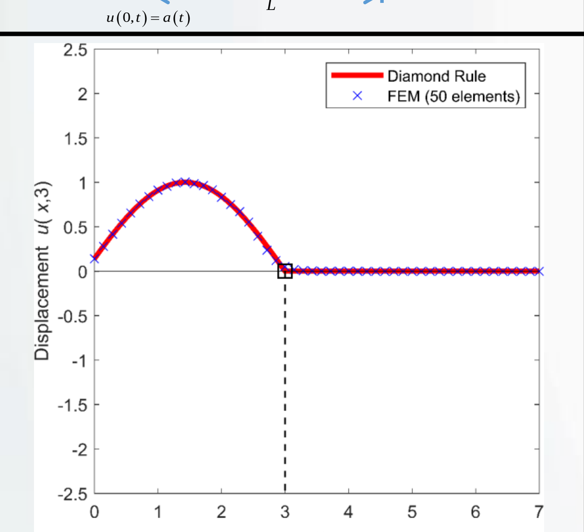
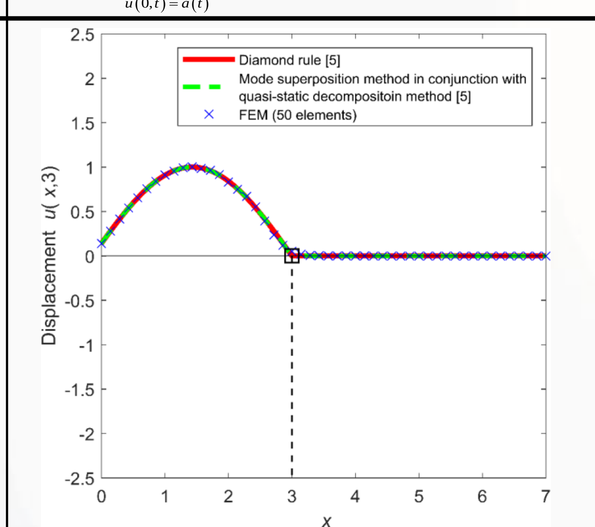
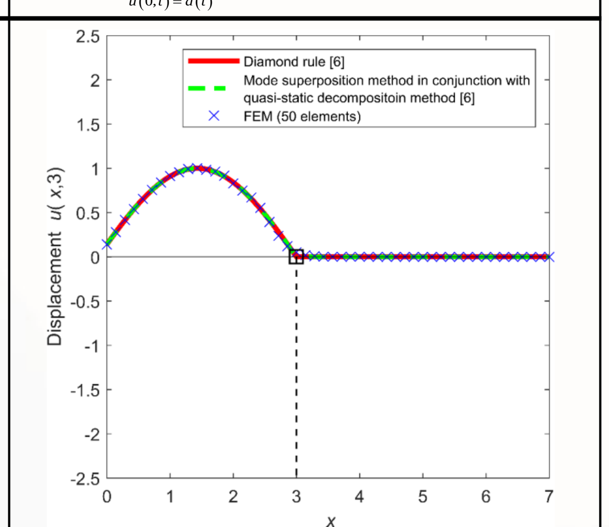
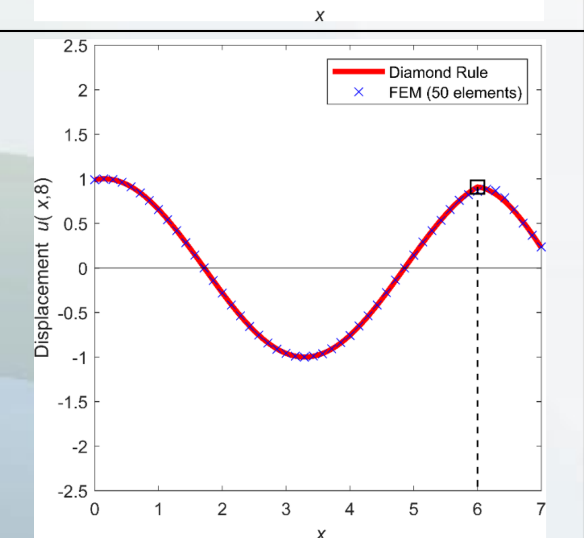
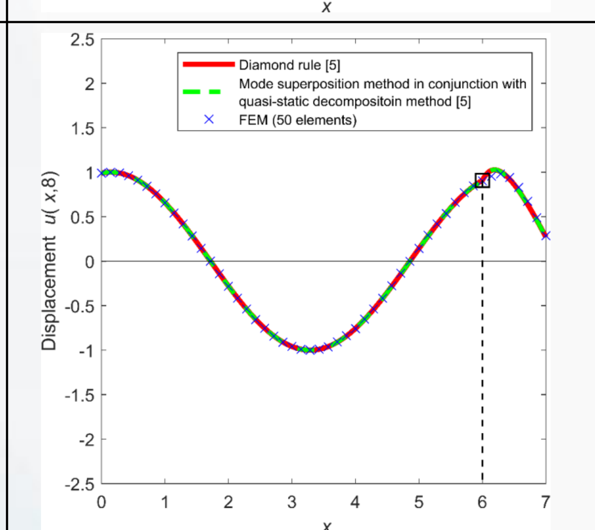
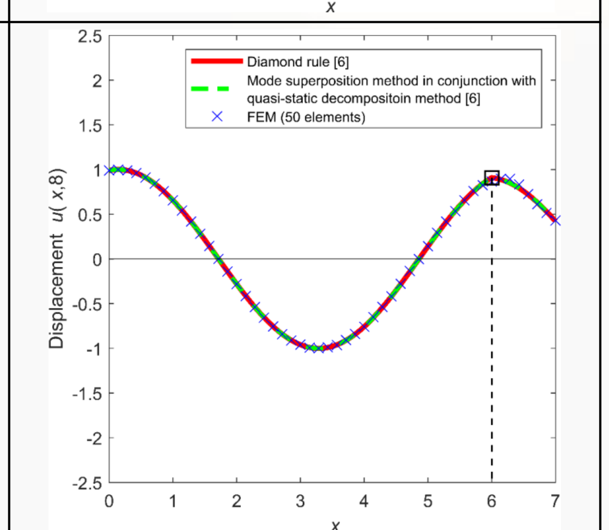
$$[\mathrm{M}]\{\ddot{\mathrm{U}}\}+[\mathrm{C}]\{\dot{\mathrm{U}}\}+[\mathrm{K}]\{\mathrm{U}\}=\{\mathrm{P}\}$$

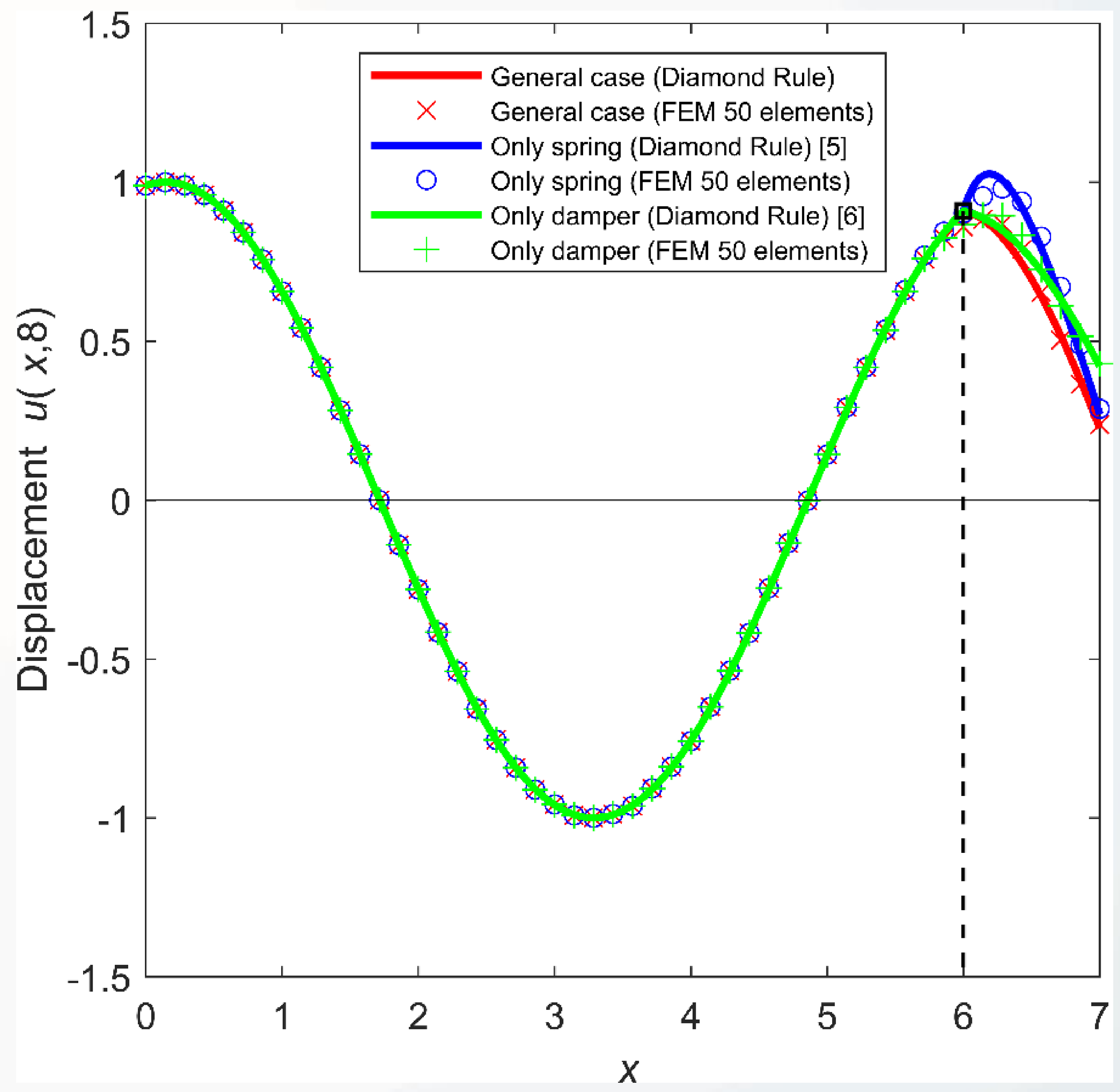
$$\begin{bmatrix}\mathrm{M}_{rr} & \mathrm{M}_{rl}\\ \mathrm{M}_{lr} & \mathrm{M}_{ll}\end{bmatrix}\begin{Bmatrix}\ddot{\mathrm{U}}_r\\ \ddot{\mathrm{U}}_l\end{Bmatrix}+\begin{bmatrix}\mathrm{C}_{rr} & \mathrm{C}_{rl}\\ \mathrm{C}_{lr} & \mathrm{C}_{ll}\end{bmatrix}\begin{Bmatrix}\dot{\mathrm{U}}_r\\ \dot{\mathrm{U}}_l\end{Bmatrix}+\begin{bmatrix}\mathrm{K}_{rr} & \mathrm{K}_{rl}\\ \mathrm{K}_{lr} & \mathrm{K}_{ll}\end{bmatrix}\begin{Bmatrix}\mathrm{U}_r\\ \mathrm{U}_l\end{Bmatrix}=\begin{Bmatrix}\mathrm{P}_r\\ \mathrm{P}_l\end{Bmatrix}$$

$$\{\mathrm{U}\}=\begin{Bmatrix}\mathrm{U}_r\\ \mathrm{U}_l\end{Bmatrix}=\{\mathrm{U}^s\}+\{\mathrm{U}^d\}=\begin{Bmatrix}\mathrm{U}_r^s\\ \mathrm{U}_l^s\end{Bmatrix}+\begin{Bmatrix}\mathrm{U}_r^d\\ \mathrm{U}_l^d\end{Bmatrix}$$

$$\{\mathrm{P}\}=\begin{Bmatrix}\mathrm{P}_r\\ \mathrm{P}_l\end{Bmatrix}=\{\mathrm{P}^s\}+\{\mathrm{P}^d\}=\begin{Bmatrix}\mathrm{P}_r^s\\ \mathrm{P}_l^s\end{Bmatrix}+\begin{Bmatrix}\mathrm{P}_r^d\\ \mathrm{P}_l^d\end{Bmatrix}$$

Results and discussion

			
Displacement profiles at t = 3 sec with the silent area			
Displacement profiles at t = 8 sec without the silent area			
Case type	General case ($c_d\neq 0,\ k\neq 0$)	Special case (Only spring) (c_d=0, $k\neq 0$)	Special case (Only damper) ($c_d\neq 0$, k=0)



Displacement profiles for three cases (spring, damper and both) at t = 8 sec

Conclusion

For the analytical and numerical methods, the method of diamond rule and the FEM were both independently employed to derive exact and numerical solutions, respectively. Our analytical approach of the method of diamond rule is both easy and simple and is highly recommended to scientists and engineers since the duration of the support motion is always short in an earthquake. For the method of diamond rule, the displacement response can be straightforwardly calculated in the space-time domain without considering the separation variables of space and time. Since the displacement was decomposed into two parts of quasi-static and inertia-dynamic solutions in the FEM, we only need few numbers of degree of freedoms, such as 50 elements, to match well with the analytical solution. This can improve the efficiency of the computer computation. In addition, the FEM can capture the dead zone or the so-called silent area. In two special cases, three approaches including the mode superposition method in conjunction with the quasi-static decomposition technique, the method of diamond rule, and the FEM yield agreeable results. However, the analytical result has not yet been derived for the general case in this study using the modal superposition method for the continuous system, since complex-valued eigenvalues and orthogonal relation cannot be expressed in an explicit form.

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