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A Variationally Consistent Reproducing Kernel Enhanced Material Point Method and its Applications to Large Deformation Problems

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Abstract

The Material Point Method (MPM) suffers from suboptimal accuracy and convergence rates as well as pressure oscillation due to the under-integration of the weak form; the material point locations with respect to the background mesh are suboptimal in performing numerical quadrature. We present an MPM framework that employs the reproducing kernel (RK) approximation to overcome pressure oscillation due to the cell-crossing instability, as well as a variationally consistent (VC) integration technique to recover optimal accuracy and convergence rates.

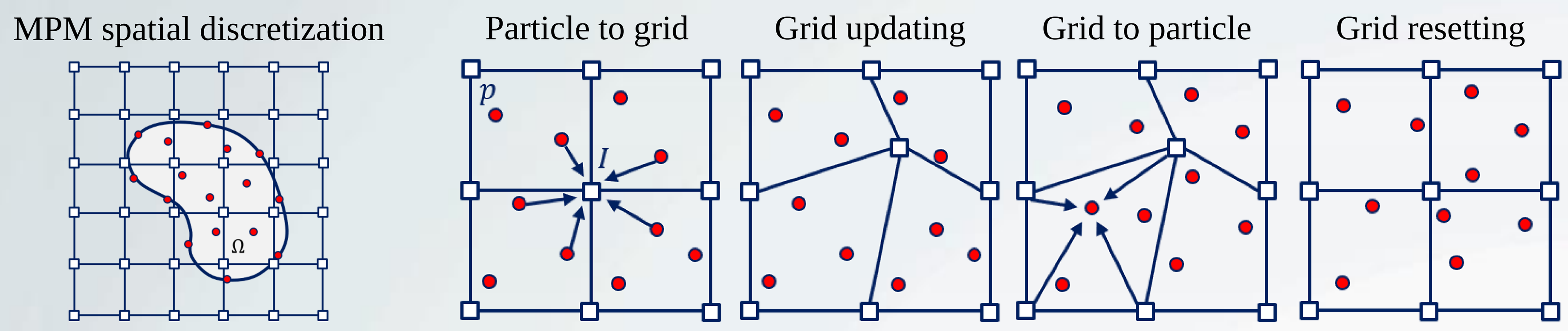
Problem Description

Material Point Method (MPM)

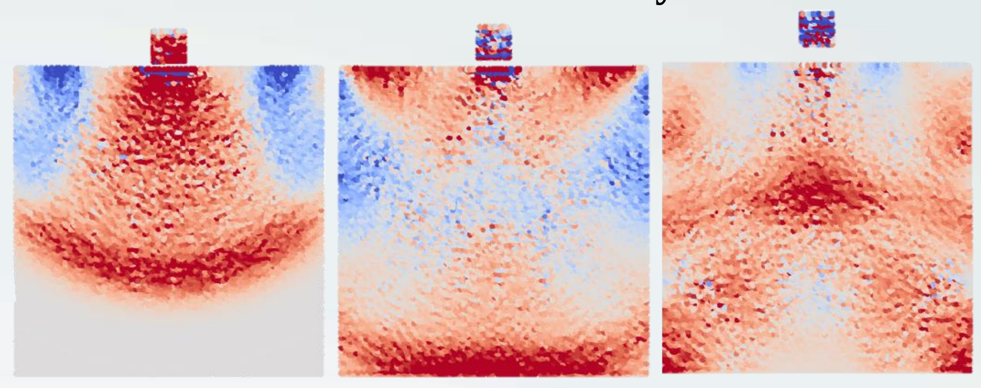
- ✓ **Eulerian** background grid mesh
- ✓ **Lagrangian** material points

Weak form for a continuum body under purely mechanical loading

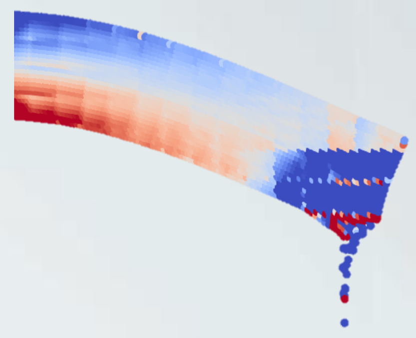
$$\int_\Omega \rho w_i^h a_i^h d\Omega + \int_\Omega w_{i,j}^h \sigma_{ij}(\boldsymbol{u}^h) d\Omega = \int_\Omega w_i^h b_i d\Omega + \int_{\partial\Omega_N} w_i^h t_i d\Gamma$$

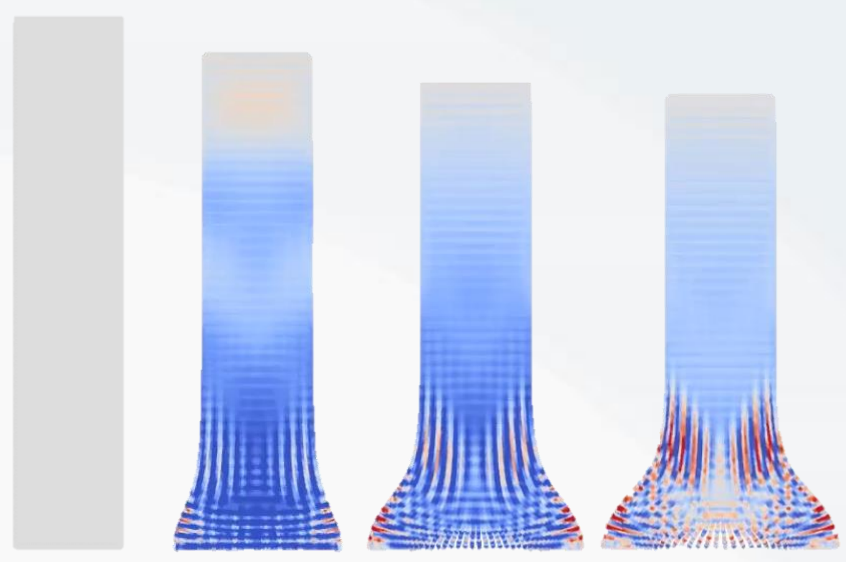


Pressure instability

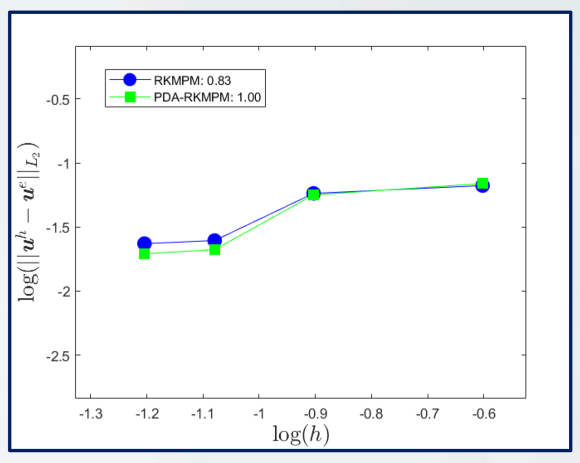


Cell-crossing instability





Suboptimal accuracy and convergence rates



Critical numerical issues:

- ❑ Linear shape functions (such as finite element) cause **stress oscillation** as material points cross cell boundaries
- ❑ The use of material point integration leads to **poor accuracy and convergence properties**
- ❑ Immature quadrature rule leads to the presence of **zero-energy modes** and **pressure oscillation**

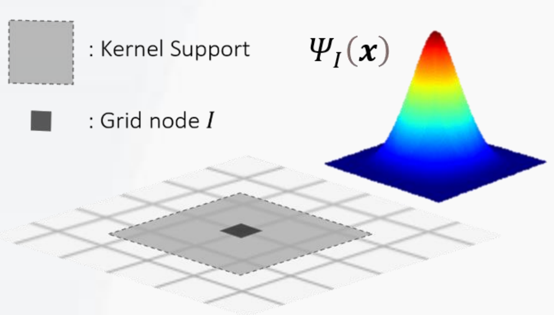
Results and Discussion

Reproducing Kernel (RK) Approximation

Displacement field approximation:

$$u_i(\boldsymbol{x}) \approx u_i^h(\boldsymbol{x}) = \sum_{I \in G_x} \Psi_I(\boldsymbol{x}) u_{iI}$$

$$\Psi_I(\boldsymbol{x}) = \boldsymbol{H}^T(0)\boldsymbol{M}^{-1}(\boldsymbol{x})\boldsymbol{H}(\boldsymbol{x}-\boldsymbol{x}_I)\phi_a(\boldsymbol{x}-\boldsymbol{x}_I)$$



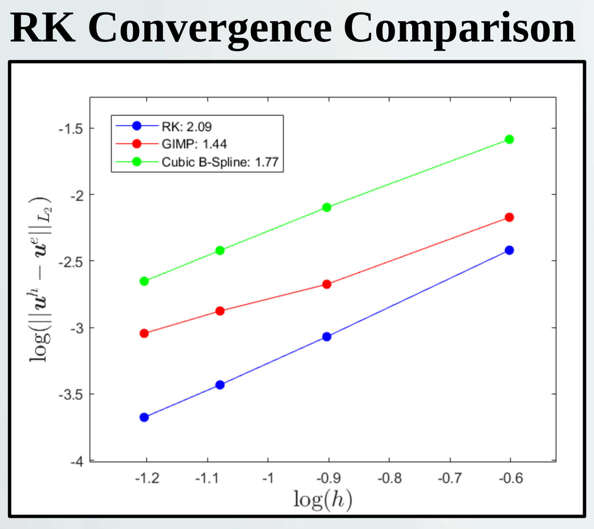
Basis vector: $\boldsymbol{H}^T(\boldsymbol{x}-\boldsymbol{x}_I) = [1, x_1 - x_{1I}, x_2 - x_{2I}, (x_1 - x_{1I})^2, \ldots, (x_2 - x_{2I})^n]$

✓ Satisfies **arbitrary order of completeness**

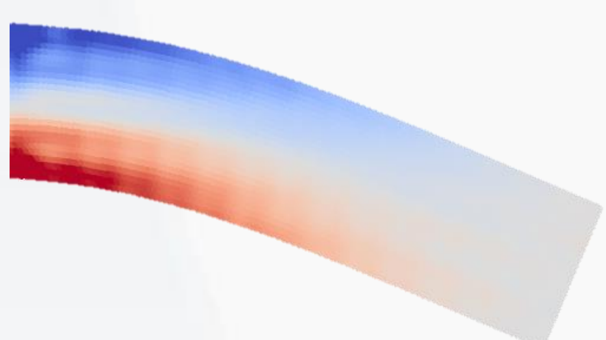
Cubic B-spline kernel $\phi_a(\boldsymbol{x}-\boldsymbol{x}_I)$

✓ Satisfies **arbitrary order of continuity**

RK Convergence Comparison



Alleviate Cell-Crossing Instability



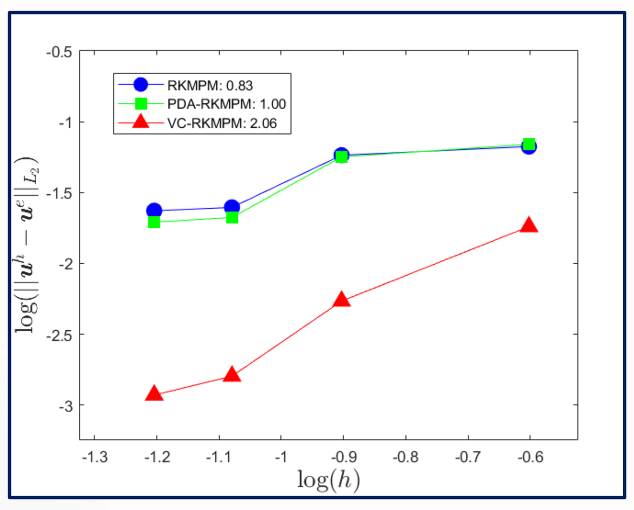
Variationally Consistent Integration

Satisfy the **divergence equality** to achieve linear **Galerkin exactness**: $\hat{\int_\Omega} \tilde{\Psi}_{I,i} d\Omega = \hat{\int_{\partial\Omega}} \tilde{\Psi}_I n_i d\Omega$

Project the **integration constraint error** into the **test function gradient field**: $\tilde{\Psi}_{I,i} = \Psi_{I,i}(\boldsymbol{x}) + R_I(\boldsymbol{x})\xi_{iI}$

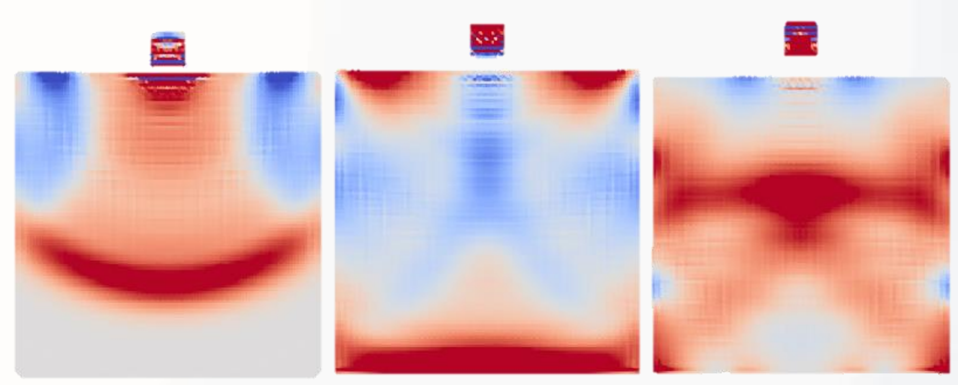
$$\xi_{iI} = -\left(\hat{\int_\Omega} R_I d\Omega\right)^{-1}\left(\hat{\int_\Omega} \tilde{\Psi}_{I,i} d\Omega - \hat{\int_{\partial\Omega}} \tilde{\Psi}_I n_i d\Omega\right) \qquad R_I(\boldsymbol{x}) = \begin{cases} 1 & \text{if } x \in supp(\Psi_I(\boldsymbol{x})) \\ 0 & \text{if } x \notin supp(\Psi_I(\boldsymbol{x})) \end{cases}$$

Optimal accuracy and convergence rates



✓ Recovers **optimal convergence**

Smoothed pressure field

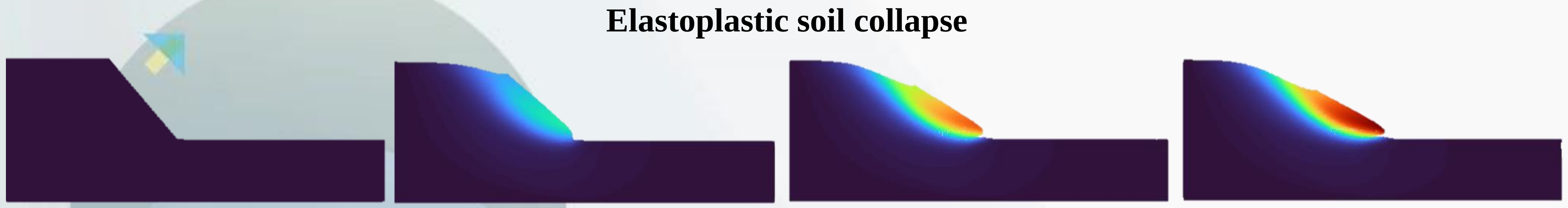


✓ **Smooths pressure oscillation**

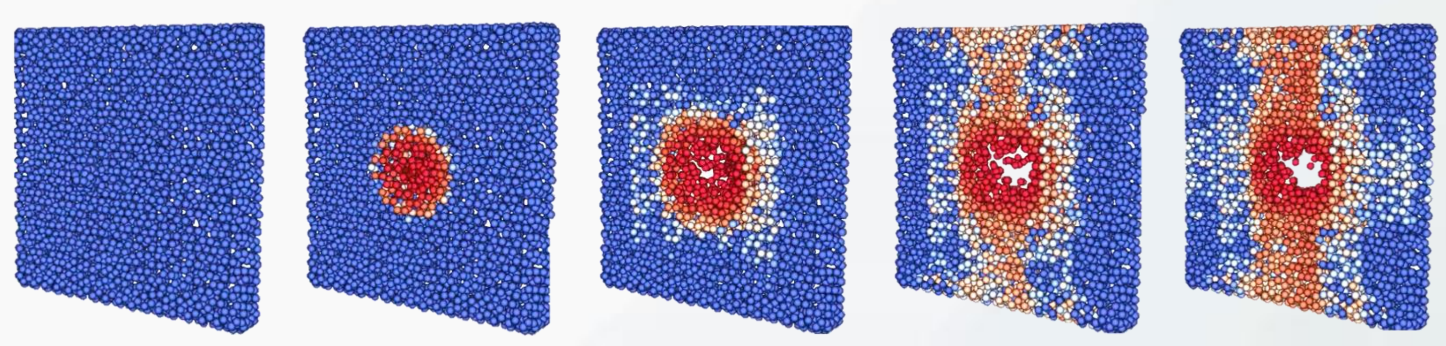
The variationally consistent MPM formulation recovers linear **Galerkin exactness**

Application to Large Deformation Problems

Elastoplastic soil collapse



Bullet penetration damage field



Conclusions

We present a Material Point Method (MPM) formulation that employs the reproducing kernel approximation to alleviate the cell-crossing instability, as well as a variationally consistent integration technique to recover optimal accuracy and convergence properties. Various numerical examples benchmark the presented MPM framework, and its application into large deformation problems involving plastic deformation as well as fracture is currently under investigation.

References

[1] D. Sulsky, Z. Chen and H. L. Schreyer, A particle method for history-dependent materials, *Computer Methods in Applied Mechanics and Engineering*, 118 (1994), pp. 179-196.
[2] J.-S. Chen, M. Hillman and M. Rüter, An arbitrary order variationally consistent integration for Galerkin meshfree methods, *International Journal for Numerical Methods in Engineering*, 95 (2013), pp. 387-418.
[3] J.-S. Chen, C. Pan, C.-T. Wu and W. K. Liu, Reproducing Kernel Particle Methods for large deformation analysis of non-linear structures, *Computer Methods in Applied Mechanics and Engineering*, 139 (1996), pp. 195-227

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