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【海報論文】

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On elastic features of tubular-architectured materials

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Abstract

Many biological materials, such as horse hooves, human teeth, and horn of bighorn ram, share the same microstructure that consists of aligned tubules which is called the tubular architecture and they perform the resistance of impact and penetration. This observation has attracted the scientists and researchers to investigate the mechanical behavior of synthetic materials with such microstructure. In this study, we attempt to explore the elastic properties of such tubular architectured materials. The unit cell approach is used with the aid of the boundary integral equation in conjunction with the degenerate kernel. Furthermore, a representation method is proposed and the analytical solution of effective elastic moduli is obtained via the analytical solutions of displacement and stress fields in the plane strain problem. According to the analytical solution of the effective elastic moduli, the influences of tubular size and matrix properties are investigated for the tubular architectured materials.

Problem description

Displacement field of Kelvin's solution [2]

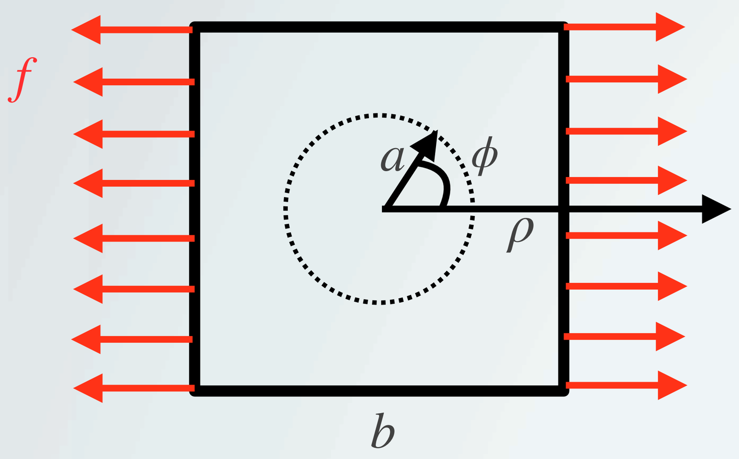
$$u_1 = \frac{(1-\nu)f}{G}\frac{a^2}{\rho}\cos\phi + \frac{f}{4G}\frac{a^2}{\rho}\left(1-\frac{a^2}{\rho^2}\right)\cos 3\phi + \frac{(1-\nu)f\rho}{2G}\cos\phi$$

$$u_2 = -\frac{(1-2\nu)f}{2G}\frac{a^2}{\rho}\sin\phi + \frac{f}{4G}\frac{a^2}{\rho}\left(1-\frac{a^2}{\rho^2}\right)\sin 3\phi - \frac{\nu f\rho}{2G}\sin\phi$$

Stress field of Kelvin's solution [2]

$$\sigma_{11} = \frac{\left[2\rho^4 - 3a^2\rho^2\cos 2\phi + a^2\left(3a^2-2\rho^2\right)\cos 4\phi\right]f}{\rho^4}$$

$$\sigma_{22} = \frac{\left[\rho^2\cos 2\phi + \left(3a^2-2\rho^2\right)\cos 4\phi\right]a^2 f}{2\rho^4} \qquad \sigma_{12} = \frac{\left[-\rho^2 + \left(6a^2-4\rho^2\right)\cos 2\phi\right]a^2 f}{2\rho^4}\sin 2\phi$$

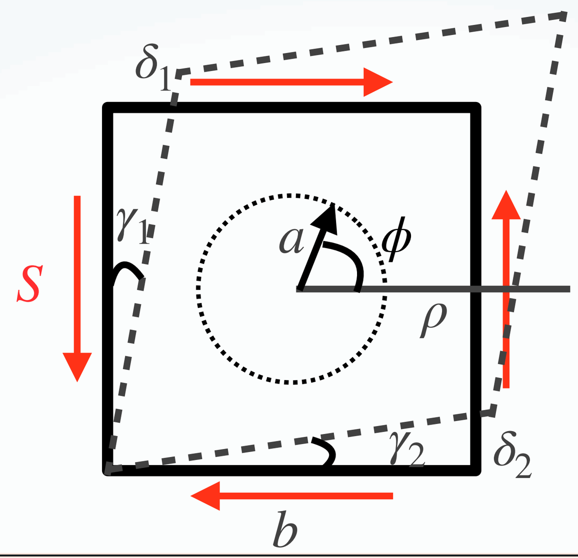


Young's modulus (1D)

$$E = \frac{\sigma}{\epsilon}$$

Global effective modulus

$$E_{eff} = \frac{\hat{F}}{\hat{u}}$$



Shear modulus (1D)

$$G = \frac{\tau}{\gamma}$$

Global effective modulus

$$G_{eff} = \frac{\hat{F}}{\hat{\delta}}$$

Results and discussion

Effective Young's modulus: $E_{eff} = \dfrac{\int_{-b/2}^{b/2}\sigma_{11}(b/2,y)dy}{u_1(\hat{\rho},\hat{\phi})} = \dfrac{4[2\hat{y} - \frac{5a^2}{4\hat{x}}\sin(4\tan^{-1}(\frac{\hat{y}}{\hat{x}})) + \int_{-b/2}^{b/2}\frac{3a^4\cos(4\tan^{-1}(\frac{y}{\hat{x}}))}{(\hat{x}^2+y^2)^2}dy]}{4(1-\nu)\frac{a^2}{\hat{\rho}}\cos\hat{\phi} + \frac{a^2}{\hat{\rho}}\left(1-\frac{a^2}{\hat{\rho}^2}\right)\cos 3\hat{\phi} + 2(1-\nu)\hat{\rho}\cos\hat{\phi}}G$

Effective Shear modulus: $G_{eff} = \dfrac{\int_{-b/2}^{b/2}\tau(b/2,y)dy}{\delta_1(\hat{\rho},\hat{\phi}) + \delta_2(\hat{\rho},\hat{\phi})} = \dfrac{2[\frac{a^2}{\hat{x}}\cos(2\tan^{-1}(\frac{\hat{y}}{\hat{x}})) + \frac{a^2}{2\hat{x}}\cos(4\tan^{-1}(\frac{\hat{y}}{\hat{x}})) + \int_{-b/2}^{b/2}(\frac{3a^4}{(\hat{x}^2+y^2)^2})\sin(4\tan^{-1}(\frac{y}{\hat{x}}))dy]}{[(\hat{\rho} + \frac{4}{1-\nu}\frac{a^2}{\hat{\rho}} - \frac{a^4}{\hat{\rho}^3}) + (\frac{2a^4}{\hat{\rho}^3} - \frac{2a^2}{\hat{\rho}})\cos 2\hat{\phi}](\cos\hat{\phi} + \sin\hat{\phi})}G$

Effective Poisson ratio: $\nu_{eff} = -\dfrac{d\varepsilon_{trans}}{d\varepsilon_{axial}} = \dfrac{2(1-2\nu)\frac{a^2}{\hat{\rho}}\sin\hat{\phi} + \frac{a^2}{\hat{\rho}}\left(1-\frac{a^2}{\hat{\rho}^2}\right)\sin 3\hat{\phi} - 2\nu\hat{\rho}\sin\hat{\phi}}{4(1-\nu)\frac{a^2}{\hat{\rho}}\cos\hat{\phi} + \frac{a^2}{\hat{\rho}}\left(1-\frac{a^2}{\hat{\rho}^2}\right)\cos 3\hat{\phi} + 2(1-\nu)\hat{\rho}\cos\hat{\phi}}$

$\nu = 0.4$	100RH%	Matrix 1	Matrix 2	40RH%
E^{exp}	140 MPa	2 GPa	3.36 GPa	5.9 GPa
$\hat{u}$(mm)	36.9	2.58	1.54	0.877

Table 1: Experimental parameters

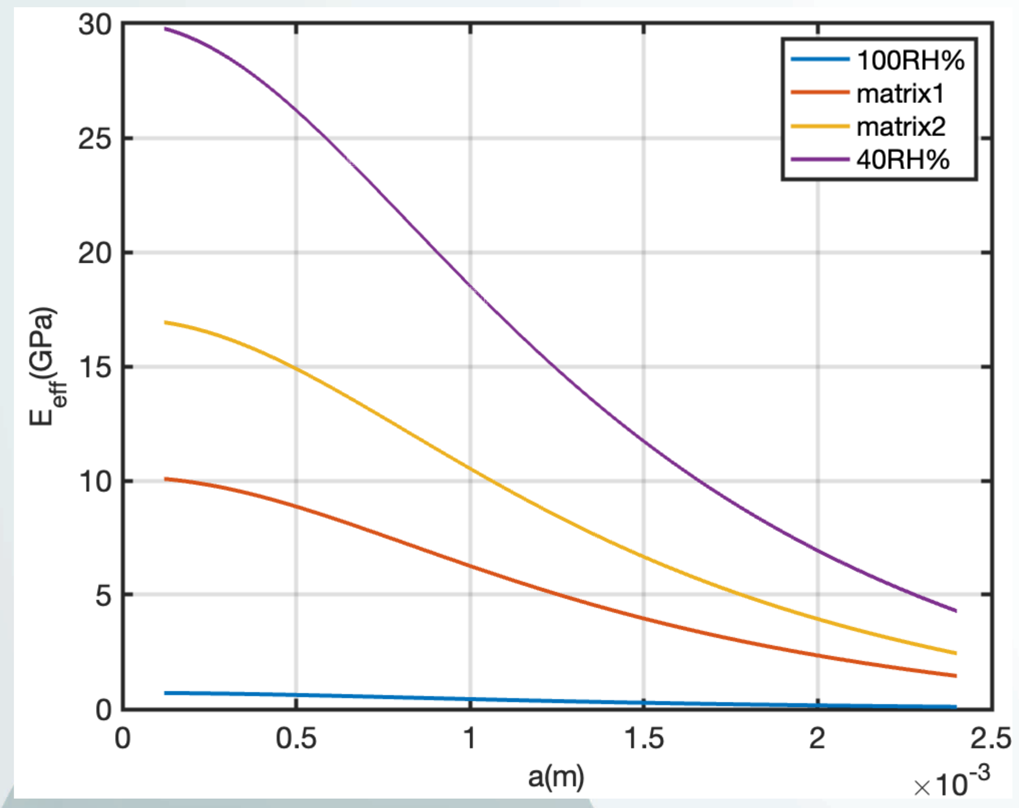


Figure 4: Effective Young's modulus with different moisture content hole size variety

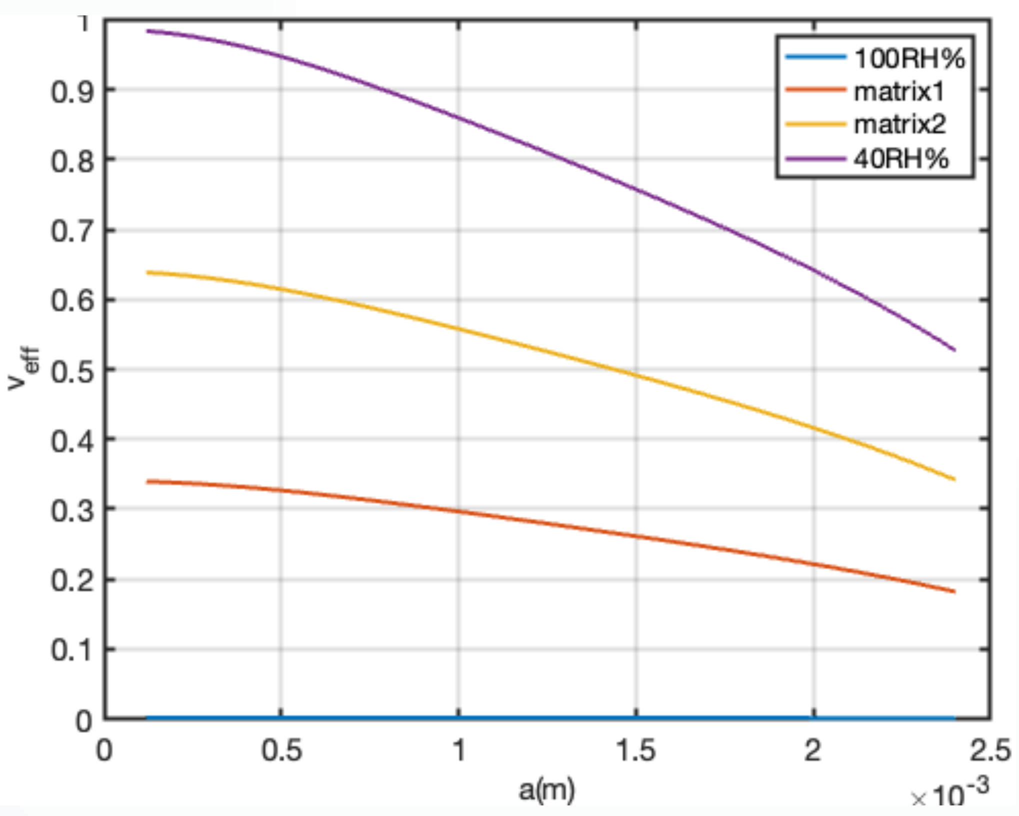


Figure 5: Effective Poisson ratio with different moisture content hole size variety

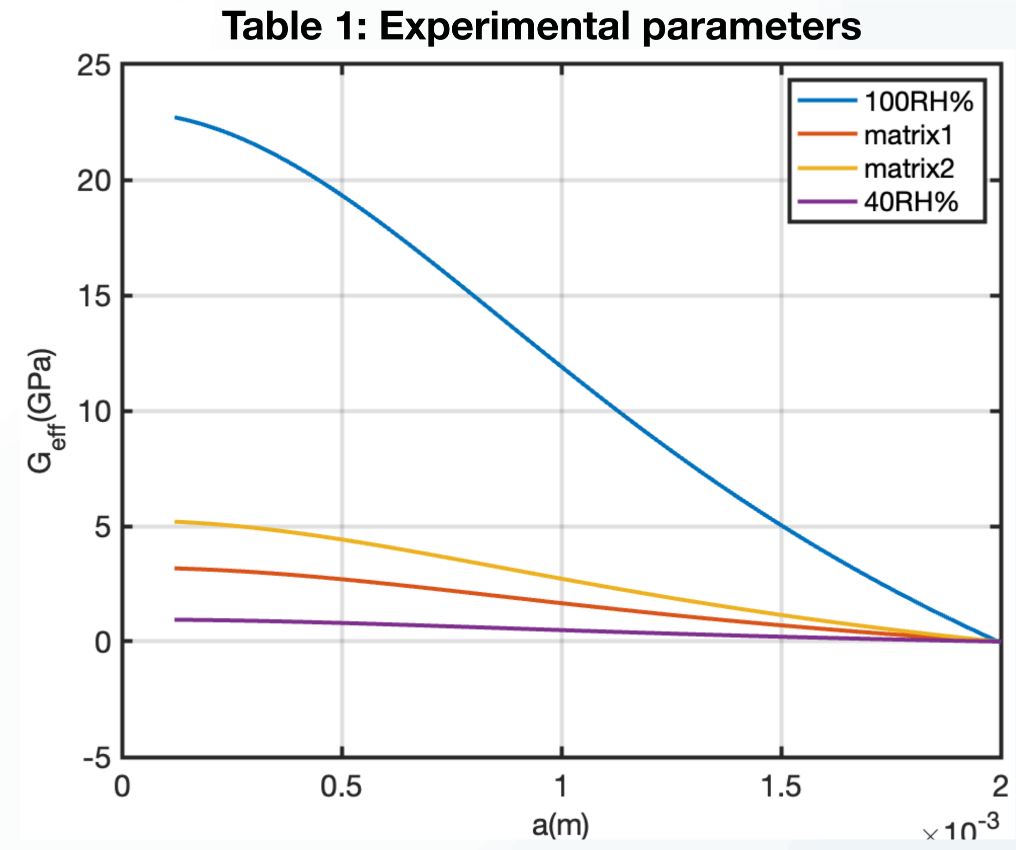


Figure 6: Effective Shear modulus with different moisture content hole size variety

Conclusions

- The analytical solution of the effective elastic moduli shows that both effective Young modulus and effective shear modulus are proportional to the shear modulus of the matrix.
- On the other hand, the effective Poisson ratio depends on the Poisson ratio of matrix merely.
- The effective elastic moduli consist of the fraction with two quartic functions of diameter of the tubule in denominator and numerator respectively.

References

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