國立臺灣師範大學 102 學年度學士班二年級轉學生招生考試試題

科目：**微積分**　　　適用學系（組）：**數學系**

注意：1.本試題共　1　頁，請依序作答，並標明題號，不必抄題。
2.答案必須寫在答案卷上之指定作答區內，否則依規定予以扣分。

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1. (a) Write out the ε - δ definition of the limit $\lim_{x\to 0} f(x)=2$.　(5 points)

   (b) Suppose that f is a continuous function on the interval (a,b), $0\in(a,b)$ and $f(0)>0$. Prove that there exists a $\delta>0$ such that $f(x)>0$ for all $x\in(-\delta,\delta)$.　(10 points)

2. Let $p, q > 1$ with $\frac{1}{p}+\frac{1}{q}=1$. Prove that $\frac{x}{p}+\frac{y}{q}\geq x^{\frac{1}{p}}y^{\frac{1}{q}}$ for all $x, y \geq 0$.　(10 points)

3. Evaluate the following integrals：

   (a) $\int_0^1 \frac{x+2}{\sqrt{4-x^2}}\,dx$　(7 points)

   (b) $\int_0^{\frac{\pi}{2}} 3^x \sin x\,dx$　(7 points)

   (c) $\int_0^{\sqrt{\frac{\pi}{2}}}\int_x^{\sqrt{\frac{\pi}{2}}}\int_0^6 \cos y^2\,dzdydx$　(8 points)

   (d) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{x^2+y^2+z^2}\,e^{-(x^2+y^2+z^2)}\,dxdydz$　(8 points)

4. Find the volume of the solid formed by revolving the region bounded by the graphs $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 3$.　(10 points)

5. Find the area of the common region bounded by the following polar curves：
   $r=-6\cos\theta$ (circle) and $r=2-2\cos\theta$ (cardioid).　(10 points)

6. Use the integral test to show that the series $\sum_{n=5}^{\infty}\frac{1}{n(\ln n)(\ln\ln n)^p}$ converges if $p > 1$.　(10 points)

7. (a) Write out the definition of a function $f(x,y)$ to be differentiable at a point (x_0,y_0).　(5 points)

   (b) Prove that if the function $f(x,y)$ is differentiable at the point (x_0,y_0), then it is continuous at (x_0,y_0).　(10 points)