

國立臺灣師範大學 103 學年度學士班二年級轉學生招生考試試題

科目：微積分

適用學系（組）：數學系

注意：1. 本試題共 1 頁，請依序作答，並標明題號，不必抄題。

2. 答案必須寫在答案卷上之指定作答區內，否則依規定予以扣分。

1. Find the limit $\lim_{x \rightarrow a} \frac{\sec x - \sec a}{x - a}$. (10 points)
2. Find the equation of the tangent line to the graph of the equation $x \sin y = y \cos x$ at the point $(\frac{\pi}{2}, \pi)$. (10 points)
3. Evaluate the following integrals:
 - (1) $\int \sqrt{1 + \sqrt{x}} dx$ (7 points)
 - (2) $\int_0^2 \frac{dx}{\sqrt{|x-1|}}$ (7 points)
 - (3) $\int \ln \sqrt{x^2 - 4} dx$ (8 points)
 - (4) $\int_0^2 \int_0^{\sqrt{4-x^2}} \sin \sqrt{x^2 + y^2} dy dx$ (8 points)
4. Find the area under the parametric curve $x = a(t - \sin t)$, $y = a(1 - \cos t)$, $0 \leq t \leq 2\pi$, $a > 0$. (10 points)
5. Find the extreme values of $f(x, y, z) = x - y + z$ on the unit sphere $x^2 + y^2 + z^2 = 1$. (10 points)
6. Integrate $f(x, y, z) = \sqrt{x^2 + y^2}$ over the involute curve $x = \cos t + t \sin t$, $y = \sin t - t \cos t$, $z = 0$, $0 \leq t \leq \sqrt{3}$. (15 points)
7. (1) Explain the Mean Value Theory of the continuous function (differential form).
No proof is needed. (5 points)
(2) Use the Mean Value Theory to prove that $\frac{31}{6} < \sqrt{27} < \frac{26}{5}$. (10 points)