國立臺灣師範大學 104 學年度學士班二年級轉學生招生考試試題

科目：微積分　　　　　　　　　　　適用學系（組）：數學系

注意：1.本試題共 1 頁，請依序作答，並標明題號，不必抄題。
　　　2.答案必須寫在答案卷上之指定作答區內，否則依規定予以扣分。

1. Evaluate the following integrals:

   (1) $\int \dfrac{x^2}{\sqrt{4-x^2}} dx$ (7 points)

   (2) $\int \sqrt{\dfrac{x}{1-x^3}} dx,\ 0 < x < 1$ (8 points)

   (3) $\int \cos(2\theta)\cos(3\theta)\,d\theta$ (7 points)

   (4) $\int_0^\infty e^{-x^2} dx$ (8 points)

2. Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{3x^2 y}{x^4 + y^2}$ or explain the limit does not exist.   (10 points)

3. Find the interval of convergence for the series $\sum_{k=1}^{\infty} \dfrac{(k!)^2}{3^k \cdot (2k)!} x^k$. (10 points)

4. Find all local maxima, local minima, and saddle points of the function

   $f(\mathrm{x},y) = \mathrm{x}^3 + 3xy + y^3$.   (15 points)

5. Find the absolute maxima and absolute minima of the function

   $g(\mathrm{x},y) = \mathrm{x}^2 + xy + y^2 - 6\mathrm{x}$ on the rectangular plate $0 \le x \le 5,\ -3 \le y \le 3$.

   (15 points)

6. Let C be a curve which is the intersection of two surfaces $x^2 + y^2 = 8$ and

   $x + z = 3$. Find the tangent line to the curve C at the point $P(2,2,1)$.

   (10 points)

7. Explain the fundamental theorem of calculus (two parts in this theorem), no
   proof is needed.   (10 points)