

國立臺灣師範大學 105 學年度學士班二年級轉學生招生考試試題

科目：微積分

適用學系(組)：數學系

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. (10%) Suppose f is continuous on $[2, 5]$ and $1 \leq f'(x) \leq 4$ for all x in $(2, 5)$. Show that $3 \leq f(5) - f(2) \leq 12$.
2. (10%) Show that, for any c , the equation $x^5 - 15x + c = 0$ has at most one root in the interval $[-1, 1]$.
3. (a). (7%) Expand $f(x) = x/(1-x)^2$ as a power series.
(b). (3%) Use part (a) to find the sum of the series

$$\sum_{n=1}^{\infty} \frac{n}{4^n}.$$

4. (10%) Find the interval on which the curve

$$y = \int_0^x \frac{1}{1+t+2t^2} dt$$

is concave upward.

5. (10%+10%) Find the limit (a) $\lim_{x \rightarrow 0^+} x^x$ and calculate (b) $\int_0^1 \tan^{-1} x dx$.
6. (10%+10%) Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a) $\int_0^4 \frac{1}{x^2 + x - 6} dx$.

(b) $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$.

7. (10%) Evaluate $\iint_D (2y - x) dA$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.
8. (10%) Evaluate $\int_C (1 + xy^2) ds$, where C is the upper half of the unit circle $x^2 + y^2 = 1$.