

# 國立臺灣師範大學 108 學年度轉系、雙主修、輔系考試試題

科目：微積分

適用系所：數學系

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

一、填充題(答案本上只寫答案，不需要寫計算過程，請標明題號)

1. (20 分) Find the following limits.

(a)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + x}}{-2x}$ ;

(b)  $\lim_{\Delta x \rightarrow 0} \frac{\sin[(\frac{\pi}{6}) + \Delta x] - \frac{1}{2}}{\Delta x}$ ;

(c)  $\lim_{x \rightarrow 0} \frac{[1/\sqrt{1+x}] - 1}{x}$ ;

(d)  $\lim_{x \rightarrow \infty} (1 + \frac{1}{2x})^x$ .

2. (15 分) Find the derivatives of the following functions:

(a)  $f(x) = \frac{6x - 5}{x^2 + 1}$ ;

(b)  $f(x) = \frac{2}{3} \sin^3 x - \frac{2}{7} \sin^7 x$ ;

(c)  $f(x) = 2x - x^2 \tan x$ .

3. (10 分) Determine the slope of the graph of

$$3(x^2 + y^2)^2 = 100xy$$

at the point (3, 1).

4. (30 分) Find the integrals. (a)  $\int \frac{x - 28}{x^2 - x - 6} dx$ ;

(b)  $\int \sin^2 \frac{\pi x}{2} dx$ ;

(c)  $\int e^{2x} \sin 4x dx$

5. (10 分) Find the power series for  $\ln(3x + 1)$  centered at  $x = 0$  and determine the interval of convergence.

二、計算題(請在答案本上寫出計算過程和答案，沒有過程不予計分)

1. (15 分) Find the volume of the solid generated by revolving the plane region bounded by the equations  $y = \frac{1}{x^4 + 1}$ ,  $y = 0$ ,  $x = 0$  and  $x = 1$  about the  $y$ -axis.