

國立臺灣師範大學 109 學年度學士班轉系考試試題

科目：微積分

適用學系（組）：數學系

注意：1. 本試題共 1 頁，請依序作答，並標明題號，不必抄題。
2. 答案必須寫在答案卷上之指定作答區內，否則依規定予以扣分。

1. Find $\frac{dy}{dx}$ in the following equations. (5 points each)

(a) $y = \ln(|\arctan(2^x)|)$ (b) $e^y \sin x = e^x \sin y$

2. Find the absolute extrema of function $f(x) = \frac{1}{2}x \ln(x^2 + 1)$ in the interval $0 \leq x \leq 2\pi$. (10 points)

3. Find the arc length of the curve given by $x = t + \sin t$, $y = 1 - \cos t$ in the interval $0 \leq x \leq 2\pi$. (10 point)

4. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n \cdot 5^n}$. (10 points)

5. Find the following integrals. (8 points each)

(a) $\int \frac{x^4 + x - 4}{x^2 + 2} dx$ (b) $\int_0^1 x^2 \ln x dx$ (c) $\int \frac{2 \cos x}{\sin^2 x - \sin x} dx$
(d) $\int_0^2 \int_0^x \int_0^{x+y} (y + 2z) dz dy dx$ (e) $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \cos(x^2 + y^2) dy dx$

6. Consider the functions $f(x, y) = 6 - x^2 - \frac{y^2}{4}$ and $g(x, y) = 2x + y$. Find a set of parametric equations of the tangent line to the curve of intersection of the surfaces at the point $(1, 2, 4)$. (10 points)

7. Determine the convergence or divergence of the following series. (5 points each)

(a) $\sum_{n=1}^{\infty} n \tan\left(\frac{1}{n}\right)$ (b) $\sum_{n=1}^{\infty} \frac{e^n}{e^{2n} + 1}$