

國立臺灣師範大學 109 學年度學士班二年級轉學生招生考試試題

科目：微積分

適用學系(組)：數學系

注意：1.本試題共 1 頁，請依序在答案本上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

一. 填充題 (答案本上只寫答案，不需要計算過程，請標明題號)

1. (20 points) Evaluate the following integrals.

(a)  $\int x \tan^{-1} x \, dx$

(b)  $\iint_R \sin(x - y) \cos^2(x + 2y) \, dA$  where  $R$  is the parallelogram on the  $xy$  plane bounded by  $x - y = 0$ ,  $x - y = \pi$ ,  $x + 2y = 0$  and  $x + 2y = \frac{1}{2}\pi$ .

2. (10 points) Find an equation of the tangent line to the curve  $e^{x+2y} - 2xy = 5$  at the point  $(2, -1)$ .

3. (10 points) Find the volume of the solid bounded above by  $z = 4 - 4(x^2 + y^2)$  and below by  $z = x^2 + y^2 - 1$ .

二. 計算證明題 (請在答案本上寫出計算或證明過程，沒有過程不予計分)

1. (10 points) Find the radius of convergence of the series  $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2} x^n$ .

2. (20 points) Find the following limits (if it exists). If the limit does not exist, explain why.

(a)  $\lim_{x \rightarrow \infty} (\ln x)^{1/x}$       (b)  $\lim_{n \rightarrow \infty} \frac{1}{n^{17}} (1^{15} + 2^{15} + 3^{15} + \dots + n^{15})$

3. (10 points) Show that the function  $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$  is differentiable at  $x = 0$  and find  $f'(0)$ .

4. (10 points) Let  $n$  be any positive integer. Show that  $f(x) = x^{2n+1} + ax - b$  cannot have two real roots for any positive number  $a, b$ .

5. (10 points) Let  $\mathbf{F}(x, y) = \left(-\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2}\right)$  and  $C$  be any piecewise

smooth simple closed curve in  $\mathbf{R}^2$  which encloses the origin  $(0, 0)$  and is oriented counterclockwise. Calculate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$