

博士班資格考試筆試各科目考試範圍與參考書目

Guidelines of Qualify Exams for admission to official candidacy for the PhD degree

實變分析 (Real Analysis)

考試範圍 (Scope) :

1. Functions of Bounded Variation and the Riemann–Stieltjes Integral
Jordan theorem, Riemann-Stieltjes integral.
2. Lebesgue Measure and Outer Measure
Algebra, σ -algebra, Borel sets, Measure spaces, Littlewood 3 principles, Cantor sets, Cantor-Lebesgue functions, Caratheodory measurable sets, Steinhaus theorem, Vitali nonmeasurable sets.
3. Lebesgue Measurable Functions
Lusin theorem, Egorov theorem, Convergence a.e. (in measure, in L^p)
4. The Lebesgue Integral
Convergence theorem (MCT, LDCT, BCT, UCT), Fatou lemma, Tchebyshey inequality, Relation between Riemann-Stieltjes integral and Lebesgue integral.
5. Repeated Integration
Fubini theorem, Tonelli theorem, Convolution.
6. Differentiation
Indefinite integral, Absolute continuous, Vitali covering lemma, Lebesgue differentiation theorem, Hardy-Littlewood theorem, Monotone functions, Convex functions, Rademacher-Stepanov theorem.
7. L^p Classes
Essential supremum, Normed linear spaces, Banach Spaces, L^p spaces, ℓ^p spaces, Separable spaces, Dual spaces, Hölder's inequality, Minkowski's inequality, Hahn-Banach theorem, Parseval formula, Bessel inequality, Complete orthonormal system, Riesz-Fischer theorem.
8. Abstract Integration
Signed measure, Additive set measure, Radon-Nikodym theorem.

參考書目 (References):

R. L. Wheeden and A. Zygmund, Measure and Integral, An Introduction to Real Analysis, Second Edition (Chapter 2,3,4,5,6,7,8,10)