

國立臺灣師範大學 113 學年度轉系考試試題

科目：微積分

適用學系所：數學系

注意：1. 本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2. 答案必須寫在指定作答區內，否則依規定扣分。

1. Evaluate the following limits. (8 points each)

(1) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

(2) $\lim_{x \rightarrow 1^+} (\ln x)^{x-1}$

(3) $\lim_{(x,y) \rightarrow (0,0)} xy \cdot \frac{x^2 - y^2}{x^2 + y^2}$

2. Find the directional derivative of the function $h(x, y, z) = x \arctan(yz)$ at the point $\mathbf{P}(4, 1, 1)$ in the direction of $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$. (10 point)

3. Find the equation of the tangent plane to the surface $z = x^2 + y^2$ at the point $(1, -1, 2)$. (8 point)

4. Determine whether the series converges or diverges, give your reason. (8 points each)

(1) $\sum_{n=1}^{\infty} (\sqrt[n]{n} + 1)^n$

(2) $\sum_{n=1}^{\infty} \ln(1 + \frac{1}{n^2})$

(3) $\sum_{n=1}^{\infty} n \sin \frac{1}{n}$

5. Find the equation of tangent line for the graph of polar equation $r = 2(1 - \cos \theta)$ at the point $(x, y) = (2, 0)$. (10 point)

6. Evaluate the following integrals: (8 points each)

(1) $\int_0^{\infty} e^{-x} \cos x \, dx$

(2) $\int_0^2 \int_x^2 2y^2 \sin(xy) \, dy \, dx$

7. Find all relative extrema of the function $f(x) = x^2 e^{-x}$. (8 points)